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ABSTRACT

Included in this set of materials is a description of an instructional model for elementary school mathematics, a curriculum guide developed by the elementary school teachers of the California Area School District, scope and sequence guides, and a teachers manual. The instructional model includes the use of pretests, posttests, prerequisites and behaviors as they relate to the idea of flexible grouping in elementary school mathematics. The teachers manual contains materials for each content item described in the curriculum guide as well as lesson plans, worksheets, prerequisites and content behaviors, pupil response sheets, enrichment materials, and individual record sheets. This work was prepared under an ESEA Title III contract. [Not available in hardcopy due to marginal legibility of original document.] (FL)

ELEMENTARY MATHEMATICS

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CALIFORNIA AREA SCHOOLS

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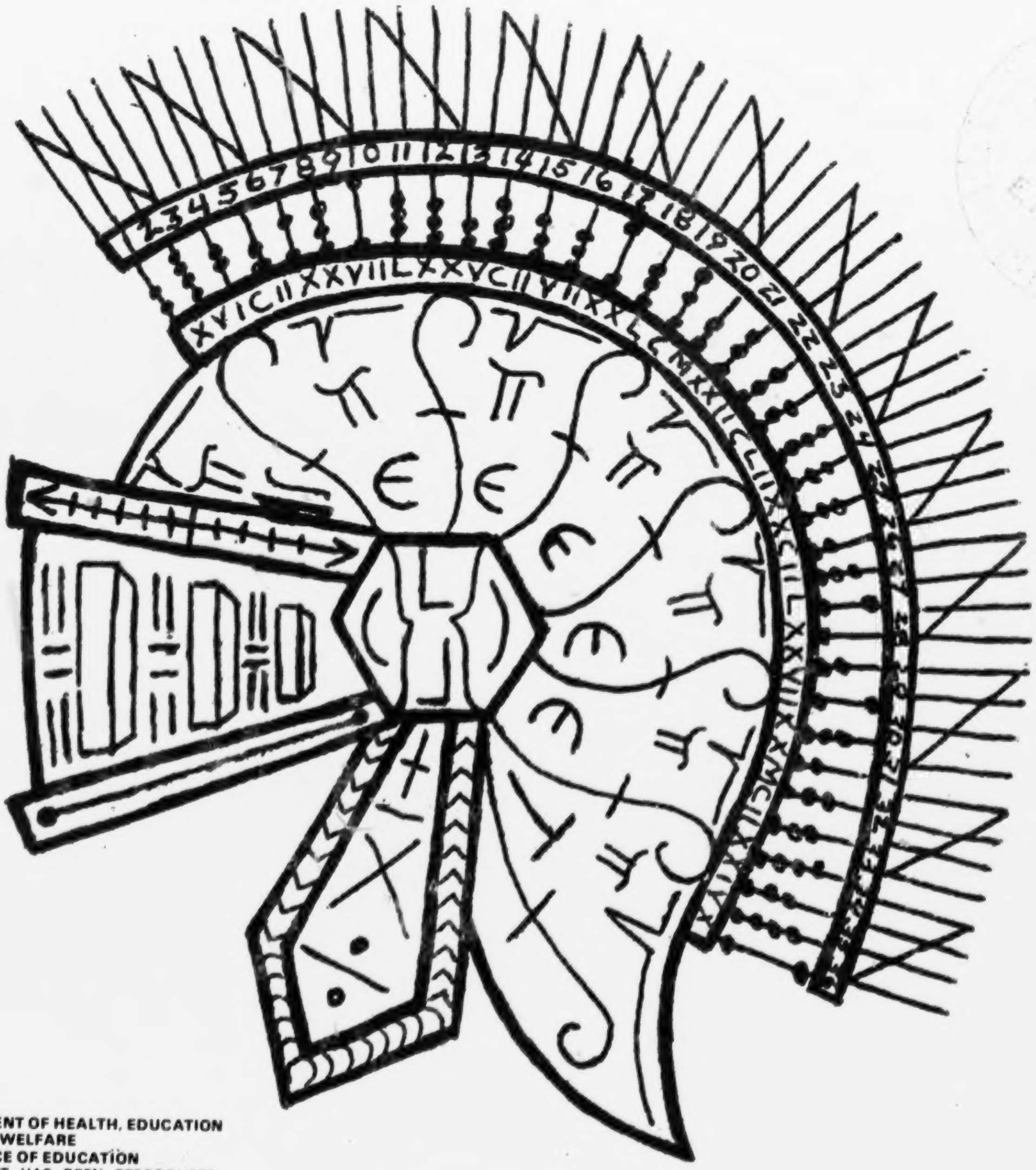
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CURRICULUM GUIDE

SCOPE & SEQUENCE GUIDE

1968 - 1969

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CURRICULUM DEVELOPMENT

ELEMENTARY MATHEMATICS

CALIFORNIA AREA SCHOOLS

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Washington, Fayette, and Greene
Counties

CURRICULUM DEVELOPMENT

ELEMENTARY MATHEMATICS

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We are also grateful for the personal assistance of the department staff, Mr. Kenneth Colkin, Miss Doris Creswell, and Mr. Emanuel Berger.

CONTENTS

Objectives of Basic Content Items

Interpretation

PRIMARY SECTION

Scope and Sequence Chart

Vocabulary

Materials

Items Ranked as to Importance

Methodology

INTERMEDIATE SECTION

Scope and Sequence Chart

Vocabulary

Materials

Items Ranked as to Importance

Methodology

OBJECTIVES OF BASIC CONTENT ITEMS

Whole Numbers

To develop a basic understanding of the structure of the system of whole numbers.

To develop the ability to work with the four fundamental processes of arithmetic.

To utilize the basic properties of associativity, commutativity, distributivity, identity, inverse, and closure.

To develop an appreciation of the use of the arithmetic of whole numbers in every day life experiences such as in problem solving, games, banking and other challenging and interesting situations.

Fractions

To develop an intuitive understanding of the concepts of fractions as they relate to real experiences.

To help students interpret the physical world they see around them.

To develop the meanings and methods of computations with fractions in all four fundamental operations.

To develop an appreciation of the value of building every mathematical process on simple, basic number relationships presented early in the students career.

Integers

To expand the students awareness of numbers other than those with which they are familiar.

To develop intuitive background of numbers less than zero in the physical environment.

To develop an understanding of integers so that they might be used in examining the property of closure for whole numbers.

To provide an opportunity for students to reason mathematically and form conclusions consistent with past experiences.

Deeper realization and appreciation of the uses and applications of integers will depend on the child as he matures in his study and understanding of basic mathematical concepts.

Numeration and Notation

To develop an understanding and deeper insight into forms of numeration and notation, such as the concept of numerals (ie other names for a number), positional notation in base ten and other bases, and symbols used in the historical evolvement of man's ideas about number.

To apply our basic notational forms to the four fundamental operations, fractions, and decimals in mathematics.

Sets

To develop concepts of, and relationships among sets.

To use sets and set notation as a basic foundation for fundamental ideas in various areas of mathematics.

To appreciate the use of sets in other content areas in the elementary school.

Geometry

To develop intuitively such concepts as point, line, plane, and space.

To explore the various interrelationships through measurement, construction and environment.

To apply geometrical ideas and patterns to the physical world.

To appreciate the artful design in geometric figures and in nature.

Measurement

To develop an understanding of measurement as a form of comparison with standard units.

To involve the students in the subject of chronology giving them some relatively simple ideas that will be useful to them in their study of history.

To develop an appreciation and an awareness of the need for measurements and standard units of measure in our society.

Number patterns and relationships

To develop an awareness of the variety of patterns in our number system.

To provide interesting and challenging situations for the growth of perceptive insight and creative approaches in the analysis of numerous mathematical arrangements.

To apply many numerical and geometric relationships encountered through classroom experiences to activities in everyday life.

To appreciate the structure and the beauty of the designs of nature.

Other topics

To provide an atmosphere conducive to ingenious and creative response to a variety of activities which extend and expand the mathematics program of the elementary school.

To appreciate the vast scope of mathematics which is involved in man's everyday activities.

INTERPRETATION

Scope and Sequence Chart

Important Items

The item number is necessary for easy access and location of content items in the Primes Content Authority List with a complete description and selected definitions.

Each grade level is divided into three month periods designated by:

B - beginning of the year
M - middle of the year
E - end of the year

In these columns will be found either

I - introduction of content item
M - mastery of content item

Thus the reader knows the grade level and approximate time of the year that a particular content item is introduced, and also the grade level and approximate time of the year when the item should be mastered.

PRIMARY

SECTION

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
0010-0100	BASIC CONCEPTS OF WHOLE NUMBER.													
0010	Definition: Set of whole numbers.					I								4
0020	Develop cardinal number zero.		I				M							
0030	Develop cardinal number 1-10.	I					M							
0035	Cardinal number beyond 10.					I						M		
0040	Develop ordinal number sense.			I							M			
0050	Number with number line.			I								M		
0060	Counting-Cardinal number of set.	I					M							
0070	Ordinal counting-sequence.		I									M		
0080	Skip counting.					I								4
0900	Backward-rote counting.		I										M	
0100	Ordering-equal, greater, less.		I											5
0110-0200	PROPERTIES OF ADDITION.													
0120	Union of disjoint sets-joining.			I								M		
0130	Developed from number line.				I								M	
0150	Commutativity.			I										4
0160	Associativity.					I								4
0170	Zero-identity element.					I								5
0180	Property of one.			I							M			
0190	Elementary fact of addition.				I						M			

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
0200	Multi-digit addition-no regrouping.					I						M		
0210	Multi-digit addition-two addends - regrouping.								I			M		
0220	More than two addends-no regrouping.					I				M				
0230	More than two addends-regrouping.								I					4
0240-0330	PROPERTIES OF SUBTRACTION-WHOLE NUMBERS.													
0250	Developed from subsets-separating.				I								M	
0260	Developed from number line.				I								M	
0270	Inverse of addition.					I								4
0280	Property of zero.				I									5
0300	Property of one.					I					M			
0310	Elementary facts of subtraction.					I								4
0320	Multi-digit subtraction-no regrouping.					I								4
0330	Multi-digit subtraction-regrouping.									I				4
0340-0520	PROPERTIES OF MULTIPLICATION-WHOLE NUMBER.													
0350	Developed from union of equivalent sets.								I					4
0360	Developed from arrays.											I		5
0370	Developed from number line.									I				4
0380	Developed as repeated addition.						I							4
0410	Commutativity.									I				5

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
0420	Associativity.												I	6
0430	Distributivity of multiplication over addition.											I		6
0440	One, the identity element.											I		4
0450	Property of zero.											I		4
0460	Elementary facts of multiplication.						I							4
0470	Multi-digit multiplication-two factors-no regrouping.											I		4
0480	Multi-digit multiplication-two factors-regrouping.												I	5
0490	More than two factors-no regrouping.												I	4
0510	Multiples or powers of ten as factors.											I		5
0530-0670	PROPERTIES OF DIVISION-WHOLE NUMBERS.													
0540	Developed from partitioning-equivalent sets.									I				4
0550	Developed as successive subtraction.											I		5
0560	Developed from the number line.											I		5
0570	Inverse of multiplication.									I				5
0575	Distributivity of division over addition.												I	6
0580	Property of one.							I						5
0610	Elementary facts of division. Divisor less than 10.											I		4

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
0620	Dividend not regrouped-no remainder. Divisor less than 10.												I	4
0630	Dividend not regrouped-remainder. Divisor less than 10.												I	5
0640	Dividend regrouped-no remainder.												I	5
0665	Powers of multiples of 10 as divisors.												I	6
0680	Solve problems-two operations.												I	5
1000-1100	BASIC NUMBER CONCEPTS WITH FRACTIONS.													
1010	Developed from subset of a given set.					I								5
1020	Developed from distances on number line.								I					6
1030	Developed from plane and solid regions.					I								6
1050	Renaming from set of fractions.											I		6
1080	Counting.							I						5
1090	Ordering: greater, less, equal to.								I					6
1110-1210	PROPERTIES OF ADDITION OF FRACTIONAL NUMBER.													
1130	Developed from the number line.											I		5
1140	Developed from plane or solid regions.							I						5
1160	Commutativity.											I		5
1180	Zero, the identity element.										I			5
1190	Common fraction notation, equal denominators.											I		5

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
1220-1310	PROPERTIES OF SUBTRACTION WITH FRACTIONS.													
1250	Developed from plane or solid regions.											I		5
1260	Inverse of adding fractions.											I		5
1270	Property of zero.											I		5
1290	Common fractions equal denominators.											I		5
1320-1440	PROPERTIES OF MULTIPLICATION WITH FRACTIONS.													
2000-2040	BASIC NUMBER CONCEPTS WITH INTEGERS.													
2020	Developed from physical world situations.											I		6
2050-2100	PROPERTIES OF ADDITION WITH INTEGERS.													
3000-3160	NUMERATION AND NOTATION.													
3000	Difference between number and numeral.				I									4
3010-3040	DIFFERENT NUMERALS FOR SAME NUMBER.													
3010	Expanded notation for whole numbers.						I							S
3015	Expanded notation for fractions.												I	5
3020	Equivalent common fractions.											I		6
3040	Other names for a number.					I								5
3050-3110	ABILITY TO READ AND WRITE.													
3050	Ten basic symbols (0-9).	I					M							
3070	Whole number units, tens (10-99).					I				M				

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
3080	Beyond tens (one hundred billion).						I							5
3090	Use commas to separate periods.											I		5
3100	Ability to round numbers for estimates.									I				6
3150	Historical systems of numeration.									I				S
4000-4125	BASIC CONCEPTS OF SETS.													
4000	Meaning: Set and members (elements).	I												5
4010	Equivalent (one-to-one correspondence).	I							M					
4020	Equal (identical).		I										M	
4030	Non-equivalent.		I						M					
4040	Unequal					I							M	
4050	One-to-many correspondence.					I								4
4060	Subsets.			I										S
4070	Empty.				I									5
4130-4170	FORMING NEW SETS BY.													
4150	Subsets.											I		6
5010-5080	RECOGNIZE GEOMETRIC FIGURES AND IDEAS.													
5010	Geometric figures in environment.	I												S
5020	Geometric designs and patterns.		I											S
5030	Relations without measure (Size, Position).		I											S

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
5040	Two-dimensional figures (plane).		I											5
5050	Three-dimensional figures (solid).								I					5
5060	Curves: simple, closed, open.	I												5
5070	Regions: interior, exterior.				I									4
5080	Representations: plane, space, point, line.	I												5
5090-5200	DESCRIBE AND ANALYZE GEOMETRIC FIGURES.													
5090	Point.								I					S
5100	Line, line segments, rays.								I					S
5130	Curves, simple, closed, open.												I	S
5180	Circles.					I								S
5190	Angles.								I					S
5210-5230	CONSTRUCT OR DRAW GEOMETRIC FIGURES.													
5210	Line constructions (segments, rays).												I	6
5234-5300	ABILITY TO MEASURE GEOMETRIC FIGURES.													
5260	Line segments (ruler, compass, other).					I								6
6000-6030	MEASUREMENT-NONGEOMETRIC QUANTITIES.													
6000	Meaning of measure.							I						S
6020	Developed of nonstandard units.											I		S
6030	English and metric units.					I								6

Item	Description	Kindergarten			First			Second			Third			Mastery Beyond Primary
		B	M	E	B	M	E	B	M	E	B	M	E	
6040-6120	ABILITY TO APPLY UNITS TO MEASURE.													
6040	Money.					I								5
6050	Time.						I							S
6060	Distance.												I	S
6070	Liquids.					I								S
6080	Temperature.					I								S
6090	Weight.											I		S
6110	Quantity (dozen, gross),						I							6
7000-7080	NUMBER PATTERNS: ELEMENTARY NUMBER THEORY.													
7000	Odd-even numbers.						I							6
7080	Rules for divisibility.												I	S
7090-7150	ABILITY TO USE NUMBER SEQUENCES.													
7090	Arithmetic progressions.											I		6
7140	Grid games.												I	S
7160	Ability to use short cuts.										I			S
8000-8150	OTHER TOPICS.													
8020	Graphing-number line.										I			S
8050	Tables, charts, graphs.												I	S
8150	Estimate and verify answers.											I		S
8160	Relations (properties of).					I								S

VOCABULARY - KINDERGARTEN

1. curve
2. line-segment
3. pattern
4. triangle
5. rectangle
6. figure
7. numeral
8. number
9. shape
10. estimate
11. measure
12. check
13. empty
14. join
15. more than
16. correspondence
17. matching
18. compare
19. identify
20. more than
21. smaller
22. larger
23. inside
24. outside
25. largest
26. member
27. ordinal
28. cardinal
29. pair
30. zero
31. number line
32. square
33. counting
34. numeral
35. closed
36. circle
37. scale
38. pattern
39. design
40. square
41. separating
42. corner
43. simple closed curve
44. set
45. subset
46. part

MATERIALS - KINDERGARTEN

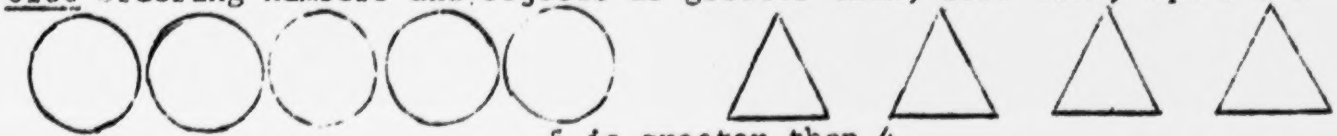
1. individual number line
2. flannelboard and stand
3. tactile numeral-blocks
4. tactile domino blocks
5. walk-on number line
6. stepping stones
7. giant beaded number cards
8. spring scale
9. pictures to illustrate lines, curves, rectangles, squares, triangles, circles
10. set of flannel numbers
11. set of flannel board objects, circle, triangle, square, and rectangle
12. figure and value matching cards
13. number learner
14. bead stair
15. shape matching cubes
16. basic rods and counters
17. Judy clock
18. number group recognition game
19. number trays and plastic counters
20. counting and threading
21. large abacus, individual abacus
22. counting discs, counting sticks
23. domino cards
24. a series of number blocks 1-10
25. number charts- 1-10

IMPORTANT ITEMS

Item	Description	Kindergarten B	M	E
0030	Develop cardinal numbers 1-10.	I		
0060	Counting-cardinal number of set.	I		
4000	Meaning: Set and members (elements).	I		
4010	Equivalent (one-to-one correspondence).	I		
4060 *	Subset			I
3050 *	Ten basic symbols (0-9).	I		
0180 *	Property of one.			I
0090	Backward-rote counting.		I	
0070 *	Ordinal counting-sequence.		I	
0050 *	Number with number line.			I
0040 *	Develop ordinal number sense.			I
0020	Develop cardinal number zero.		I	
4030	Non-equivalent sets.		I	
0150 *	Commutativity-whole number.			I
0120 *	Union of disjoint sets-joining.			I
0100 *	Ordering-equal, greater, less.		I	
4020	Equal sets (identical).		I	
5080	Represent line, point, plane.	I		
5010	Geometric figures in environment.	I		
5030	Relations without measure (size, position).		I	
5040	Two-dimensional figures (plane).		I	
5020 *	Geometric design and pattern.		I	
5060	Curves: simple, closed, open.	I		
9050	Review: numeration.			I
	* Difficult Items			

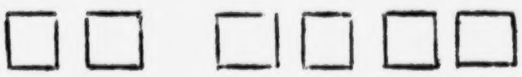
METHODOLOGY - KINDERGARTEN

0100 Ordering numbers and objects as greater than; less than; equal to.

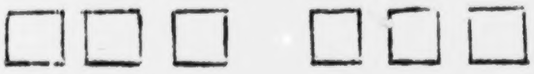


5 is greater than 4

The teacher explains that the number of circles is greater than the number of triangles and that 5 is greater than 4. The children must have the concept of a numeral representing the number of objects in a given collection matching the circles and triangles in one-to-one correspondence, demonstrates that there are more circles than triangles therefore the cardinal number of the set of circles is greater than the cardinal number of the set of triangles.

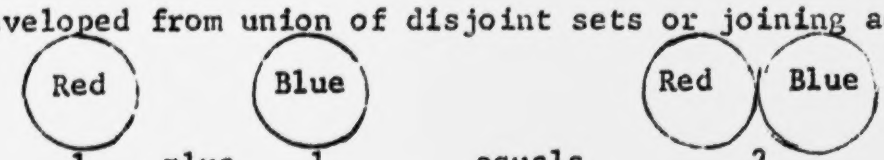


2 is less than 4



3 is equal to 3

0120 Addition developed from union of disjoint sets or joining action.

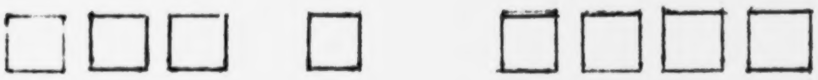


1 plus 1 equals 2

The children should have a basic understanding of sets and elements in a set. Place one cutout on the left side of the flannel board and another (different color or shape) cutout on the other side of the board. Have the children name the number of elements in each set. Have a child join the two sets by moving one set over next to the other set. Note that when two sets of one object each are joined, a new set containing two objects is formed. The sentence illustrates that one plus one and two name the same number. When the two sets are joined, a new set with more members is formed. Introduce the term "addition" as being the mathematical word for combining two numbers, using this joining action of sets for development. Introduce "plus" and "equal", emphasizing that we use these words only with numbers, not with sets.

0130 Addition: Property of one

A set of 1 object will be joined to a set of 3 objects and a new set of 4 objects will be formed.

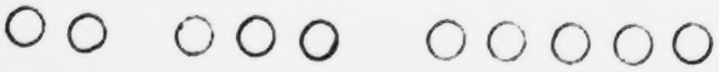


3 objects 1 object 4 objects

Bring 3 children to the front of the room ask how many there will be if one more child joins the group. When there are 4 children up front, ask how many there will be if one more is included. Continue in this manner with other examples emphasizing that when we add one to any number our sum is the next number when we are counting.

3 + 1 = 4 4 + 1 = 5 5 + 1 = 6

0150 Commutativity, a property of addition

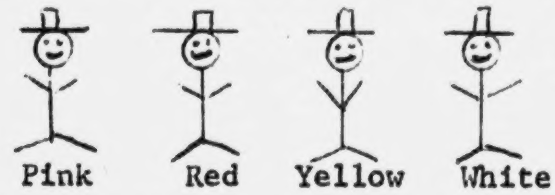


2 + 3 = 5

3 + 2 = 5

Have two boys come to the front of the room; then have three more boys join them. Ask how many we have when we join a set of 2 boys and a set of 3 boys or what number 2 + 3 is. Follow the same procedure, but have the 3 boys up front first and have 2 boys join them. Emphasize that we may join sets or we may add numbers in any order.

0070 Ordinal counting for sequence of numbers.

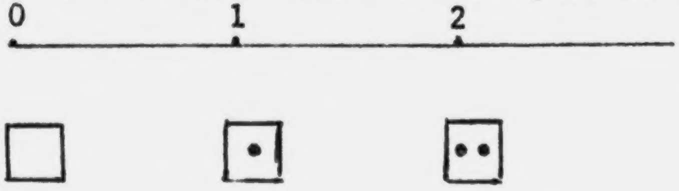


Place different colored hats on children's heads.

Which child has the pink hat on his head, the red hat, the yellow hat, etc.?

Have the children line up and pretend to be buying tickets. Discuss the position of each child from some starting point and relate to first or second etc. Hence first, second, their etc. are ordinal numbers. Extend the lesson to various series of things in the classroom, for example series of pictures on the bulletin board. Have the children hold cards with the numbers in order and discuss first, second, third, etc.

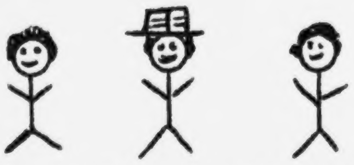
0500 Associating idea of number with number line. (one-to-one correspondence)



On the board mark two points. Connect the two points to form a part of a line or a line segment. Introduce the phrase "part of a line" has points to show where it ends. Then on a straight line join another "part" and ask how many parts of a line are there now. Draw ten segments. Ask children, "If you were going to use numerals to name the points, what numeral would show the first point with no parts on the line." Label it zero. Then proceed to the next point and ask how many parts of a line are there from zero to the point (mark the point for one with your finger)? Then ask the children to select a number to describe a single part of the line, and continue in this manner.

0040 Developing ordinal number sense.

Line up 3 children in front of the class.



Which child has a hat on his head to make him different? What is the child's position in the line ? Ordinal number indicates order or position and tells which one. See 0070, ordinal counting.

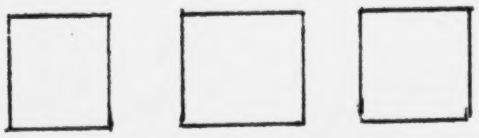
3050 Read and write nonnegative whole numbers for the ten basic sumbols (0-9).

Begin with a bare table and ask "How many objects are on the table?" (Accept "none" as an answer.)

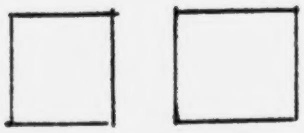
The empty set has no members. The set with no members, called the empty set, has a cardinal number, zero.

Place a single object on the table and ask, "How many objects are now in the set?" Introduce the notation card 1 at this time. Continue to place objects on the table, repeat the same quest, and introduce the notation card for other numbers such as 2, 3, etc.

4060 Subsets



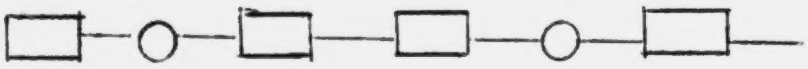
Red blocks



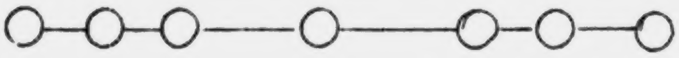
Yellow blocks

Begin by placing the above objects on a table. Explain that even though these blocks have different colors they form a set of five blocks. Remove the yellow blocks. These yellow blocks form a set of two blocks. The yellow blocks form a subset of the set of 5 colored blocks. Similarly, the red blocks form a subset of the set of 5 colored blocks.

5020 Geometric designs or patterns



Using beads of 2 different shapes but of the same color, start a string of beads. Alternate the 2 different shapes. When the string is long enough, point out the sequence of beads - explain that this is a pattern. Have a child continue the pattern. Start another bead pattern and ask which beads should be used next to continue the pattern.



The teacher may have the child make a pattern of his own.

VOCABULARY - GRADE 1

1. number
2. numeral
3. set
4. fewer
5. greater than
6. less than
7. add
8. addends
9. equals
10. equations
11. join
12. minus
13. missing addends
14. plus
15. separate
16. solve
17. subtract
18. sum
19. take away
20. amount
21. contains
22. family of facts
23. grouping
24. half-past
25. o'clock
26. order
27. order rule
28. pint
29. quart
30. rename
31. abacus
32. bundles of ten
33. compare
34. parentheses
35. subset
36. subtraction
37. addition
38. boundary
39. closed curve
40. curve
41. fold line
42. fraction
43. line
44. line segment
45. measure
46. outside
47. plane
48. point
49. rectangle
50. region

51. side

52. unit

53. place value

54. opposite

55. regrouping

56. even

57. odd

58. empty set

59. elements

60. whole number

61. pologon

62. equivalent sets

63. non-equivalent sets

64. inch

65. foot

66. degree

67. match

68. cup

69. one half

70. subset

71. multiplication

72. factors

73. product

74. times

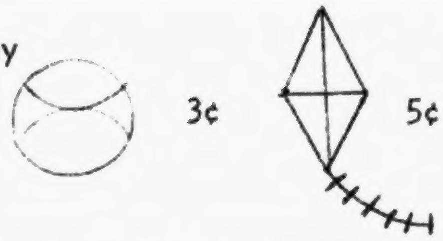
MATERIALS - GRADE 1

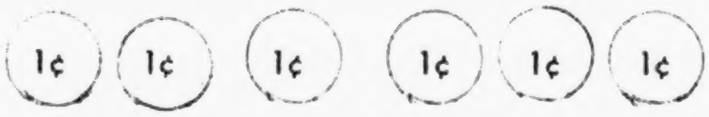
1. teacher model abacus
2. number concepts
3. visual fractions
4. time learner
5. Judy clock
6. aluminum liquid measures
7. individual number lines (0-20 or higher)
8. plastic counting rods
9. geometric shapes-squares, triangles etc.
10. individual abacus
11. flannel 3" numbers
12. flannel 7" fractional circles
13. flannel 7" fractional squares
14. count-a-ladder

IMPORTANT ITEMS

Item	Description	Grade 1 B	 M	 E
0060	Counting-cardinal number of set.			M
0030	Develop cardinal number 1-10.			M
3050	Ten basic symbols (0-9).			M
0190	Elementary facts of addition.	I		
0130	Addition developed from number line.	I		
0250 *	Subtraction developed from subsets.	I		
0260 *	Subtraction developed from number line.	I		
0310	Elementary facts of subtraction.		I	
0270 *	Inverse of addition-subtraction.		I	
0035 *	Cardinal number beyond 10.		I	
3070	Whole number units, tens (10-99).		I	
0300	Property of one-subtraction.		I	
3080	Units beyond tens.			I
0220 *	More than two addends-no regrouping.		I	
0200 *	Multi-digit addition-no regrouping.		I	
0320	Multi-digit subtraction-no regrouping.		I	
3040	Other names for a number.		I	
0160 *	Associativity of addition.		I	
4040	Unequal sets.		I	
0170 *	Zero-identity element.		I	
0280 *	Property of zero-subtraction.		I	
4050	One-to-many correspondence.		I	
4070	Empty set.	I		
6040 *	Units of measure-money.		I	
3000	Difference between number and numeral.	I		
	* Difficult Items			

6060 Money





How many balls can you buy for the amount of money shown?

After the children has a good understanding of prices shown and money values this type exercise could be given. Have the children put the money into sets according to the cost of the item in question then count the number sets. For example, since the item in question is the ball which costs 3 cents, the coins should be put into sets of 3 cents which would give us 2 sets. Therefore, for the amount of money shown 2 balls can be bought. Practice using money in "playstore" situations could prove quite useful in developing background for use of money.

0270 Subtraction is the inverse of addition.

$\square + 3 = 11 \qquad 11 - 3 = \square$

$8 + 3 = 11 \qquad 11 - 3 = 8$

Introduce this type of problem using sets of concrete items. For example use 8 girls and 3 boys. Show than when we bring the 3 boys to join the 8 girls we have 11 children. Explain that by subtracting the 3 boys from the set of 11 children we will "undoing" what we have just "done" and that we will end up with the original set of 8 girls. Thus when we know the sum and one of the addends, the missing addend can be found by subtracting the known addend from the sum. The child will know that 8 + 3 = 11, is one of the basic addition facts he must solve for the missing portion in this problem.

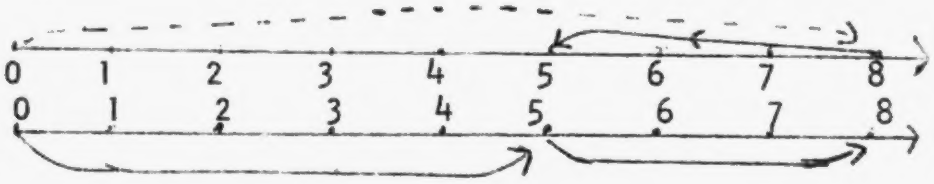
$\square + 3 = 11$ or $8 + \square = 11$

Thus we establish the idea that $\square + 2 = 5$ is the inverse of $3 + 2 = \triangle$

and that $\square + 2 = 5$ is the same as $5 - 2 = \square$

0260 Subtraction from number line.

$8 - 3 = \square$



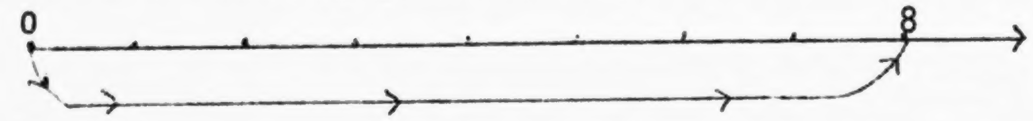
The teacher should review some basic ideas of addition on a number line such as:

$5 + 3 = \square$

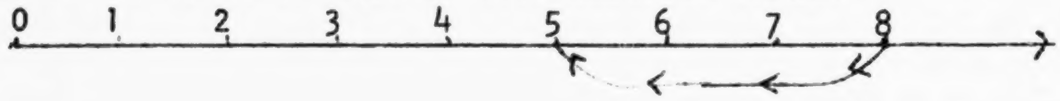
Here the child should note that addition of 2 numbers also implies a direction in traveling along the number line (in this case we moved to the right).

Now we can relate subtraction to the inverse of addition. (See code no. 0270)

For the problem 8 - 3 = ▭ , the child must first move 8 spaces from the starting position, zero.



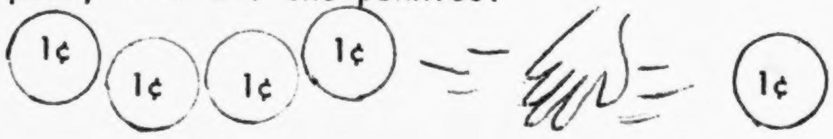
Since subtraction is the inverse of addition, we now move 3 spaces to the left, so that we will subtract three.



Our final stopping position illustrates the end result five. It also can be related to "take away" as used in subtraction. The teacher may use the number line with games, such as "traveling", "taking trips", etc.

0250 Subtraction developed in relation to subsets or separating action.

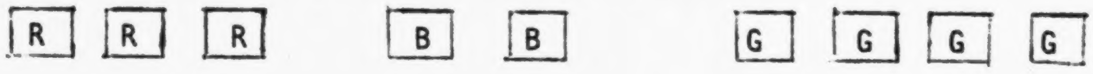
Separate one penny from all the pennies.



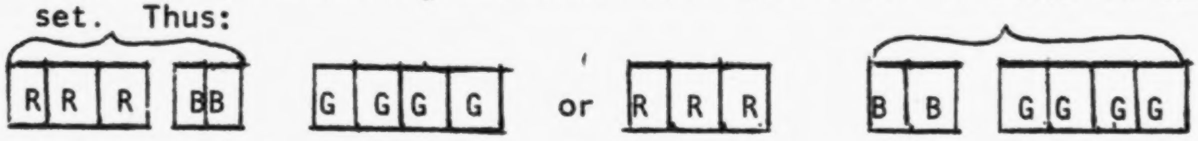
5 - 1 = 4

Use sets of concrete items or flannel board objects to show this concept. For example put the five pennies on the desk then "remove" or "separate" one penny from the set and see how many are left of the original set. Continue practice using objects to show separation.

0160 Associativity, a property of addition.



Using objects, demonstrate that it makes no difference in which manner we group the sets together we will end up with the same set of objects. For example if we have 3 red books, 2 blue books, and 4 green books we can group the red and the blue books first then join the green books to that set, or we can group the green and blue books then join thered books to that set. We can relate this grouping of books to the associativity property of addition by concerning ourselves with the number of books in each set. Thus:



$$(3 + 2) + 4 = 3 + (2 + 4)$$

$$5 + 4 = 3 + 6$$

$$9 = 9$$

0170 Zero, the identity element of addition.

$n + 1 = 1 + n = n$
$4 + 0 = 0 + 4 = 4$

After the children have mastered the concept that 0 is the name given to the empty set, the concept of the identity element can be introduced. Show through the use of objects that regardless of which set we join to an empty set we will have that original set. Show also that if we add the identity element (zero) to any number we will have that number. After these concepts are shown through the use of sets it can also be shown through the use of the number line using zero to denote "not any" steps.

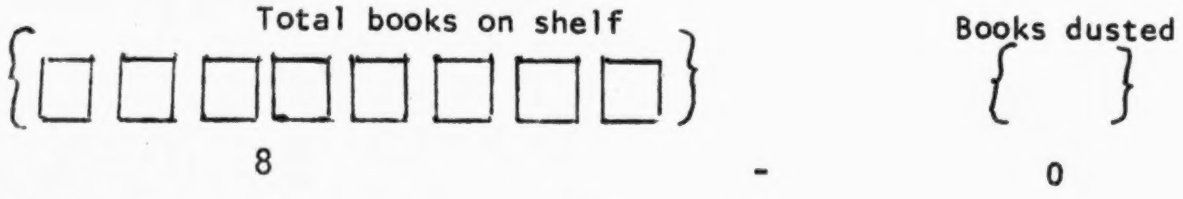
0280 Property of zero in subtraction.

$n - n = 0 \qquad 8 - 8 = 0$

$n - 0 = n \qquad 8 - 0 = 8$

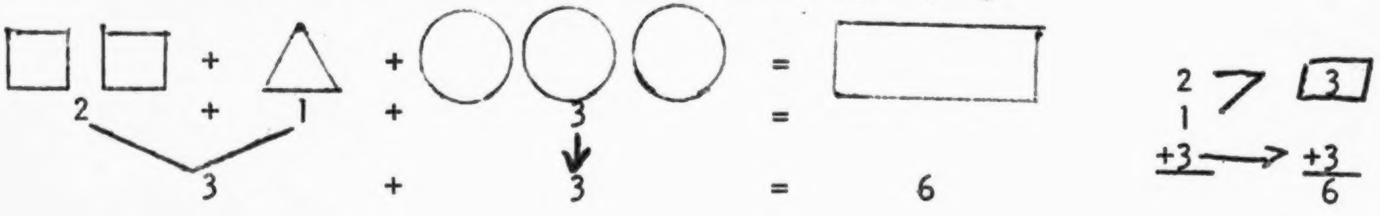
Children should have the understanding that zero represents the empty set and "not any" steps on the number line. If we start at any point, moving "0" steps on the number line keeps us at the same place. See subtraction from number line.

John has the task of dusting the set of books on the top shelf in the library. John cannot reach this particular shelf, so the set of books which he dusted on the top shelf is the empty set. Since the top shelf has eight books in the set and the number of books dusted is zero, if we separate the books dusted from the books on the shelf, there are still eight books to be dusted.



= 8

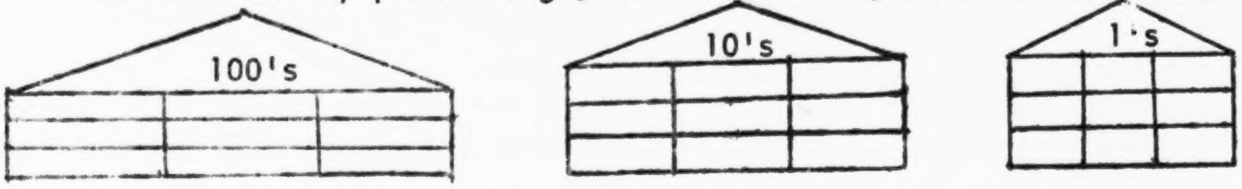
0220 Addition with more than two addends, without regrouping.



In the addition of 3 addends it must be pointed out that first two addends must be added then the third addend added to that sum. For example in the problem 2 + 1 + 4 = 7 we would first add the 2 and 1 then add the sum 3 to the third addend 4 for a total of 7. The associative property of addition could also be introduced by showing that the 2 and 4 can be added then the 1 added to that sum. The teacher may use objects to illustrate joining the sets two at a time.

0035 Developing cardinal numbers beyond 10.

Develop the idea of place value in the number system. This can be done by presenting 3 "houses" with 9 rooms in each house.

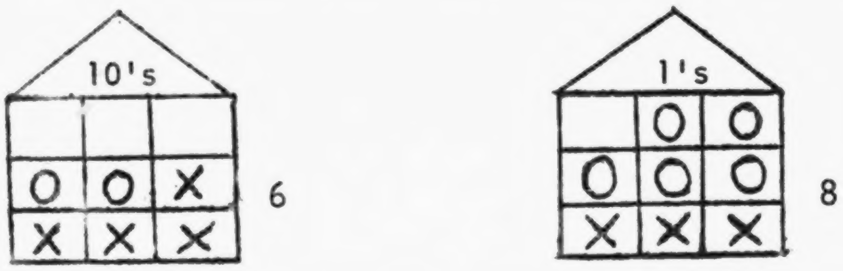


Explain that we can put 1 stick, straw, etc. in each room of Mr. One's place until we get 9 then if we want to add 1 to more we must move the whole "family" of 10 units into 1 apartment of Mr. Ten's place. To find the numeral that represents the number we have, we must count the number of rooms filled in Mr. Ten's place, and the number of rooms in Mr. One's place. Proceed in this manner, moving one more into Mr. One's place and writing the numeral to represent the number until we get to 100. The same procedure may be followed to write numerals in the hundreds. Eventually, the children should then be able to write the numerals without the "houses." During this exercise the numbers should be named in terms of "tens" and "ones." The conventional names can then be presented to the children in decades, (ex. twenties, thirties, etc.).

0200 Multi-digit addition without regrouping.

$$\begin{array}{r} 43 \\ +\underline{25} \end{array} \qquad \begin{array}{r} 78 \\ +\underline{10} \end{array}$$

After the children know the concept of the tens place and the ones place and understand the concept of joining sets as a basic step for developing the addition, then explain that in the addition of two place addends, we add the ones first then we add the tens. This concept may be developed through the use of the "houses" again.



43 will be 4 x's in the tens house and 3 x's in the one house. 25 will be 2 O's in the tens house and 5 O's in the ones house. Count the number of rooms filled in each house.

Insist that the children add the ones first then add the tens. This is important so that the concept of regrouping may be more easily introduced.

VOCABULARY - GRADE 2

1. add-addend
2. missing addend
3. cardinal number
4. digit
5. doing-undoing
6. equals
7. equation
8. empty set
9. Family of Facts
10. form
11. greater
12. greater than
13. less than
14. how many
15. join
16. less
17. longer
18. match
19. match one-to-one
20. name for a number
21. number line
22. plus
23. sum
24. numeral
25. remove
26. left
27. right
28. separate
29. set
30. shorter
31. subtract
32. subtraction
33. minus
34. take away
35. to the left of
36. to the right of
37. vertical
38. abacus
39. grouped
40. grouping
41. regrouping
42. number pattern
43. one more
44. order
45. pairs
46. paired number lines
47. parentheses
48. pattern
49. curve
50. end point

51. foot
52. half hour
53. hour
54. line
55. line segment
56. measure
57. one half
58. pint
59. point
60. quart
61. standard units
62. unit
63. ones
64. ones' place
65. set of 10
66. tens' place
67. cent
68. dime
69. half-dollar
70. nickel
71. making change
72. penny
73. quarter
74. value
75. circle
76. closed
77. complex
78. corners
79. fraction
80. gallon
81. inside
82. not closed
83. not simple
84. one fourth
85. two fourths
86. three fourths
87. one third
88. two thirds
89. outside
90. rectangle
91. region
92. sides
93. simple closed curve
94. square
95. triangle
96. grouping property
97. hundred
98. renaming
99. tens
100. one ten

101. ten tens

102. ten ones

103. thousand

104. value

105. equivalent

106. even

107. odd

108. factor

109. multiple

110. multiplication

111. multiply

112. product

113. times

114. one greater than

115. whole numbers

116. coin

MATERIALS - GRADE 2

1. tactile numeral board
2. folding black flannel board
3. minute minder
4. primary cutouts
5. number line - teacher and individual
6. abacus - teacher model
7. colored counting cubes
8. individual student abacus
9. colored rods
10. counting slide rule
11. time learner
12. visual fractions
13. Judy clock
14. counting disks-500, red
15. analysis of one foot
16. aluminum liquid measures
17. geometric shapes - squares, triangles, etc.
18. count-a-ladder

IMPORTANT ITEMS

Grade 2

Item	Description	B	M	E
3070	Read and write whole number units 10-99.			M
0020	Develop cardinal number zero.			M
4010 *	Equivalent sets (one-to-one correspondence).		M	
4030	Non-equivalent sets.		M	
0210	Multi-digit addition-two addends.		I	
0220	More than two addends-regrouping.			M
0230	More than two addends-regrouping.		I	
0350 *	Multiplication developed from union of equivalent sets.		I	
1140	Addition of fractions-developed from regions.	I		
1090 *	Ordering: greater, less, equal.		I	
0410	Commutativity-multiplication.			I
0570 *	Division-Inverse of multiplication.			I
0540 *	Division developed from equvalent sets.			I
6000	Meaning of measure.	I		
0330 *	Multi-digit subtraction-regrouping.			I
0370 *	Multiplication developed from number line.			I
1020 *	Fractions developed from number line.		I	
5050	Three-dimensional figures.		I	
3100 *	Ability to round number for estimates.			I
5100	Describe and analyze-line, ray.		I	
5090	Describe and analyze-point.		I	
5190 *	Describe and analyze-angles.		I	
3150	Historical systems of numeration.			I

* Difficult Items

METHODOLOGY - GRADE 2

0350 Multiplication developed from union of two or more equivalent sets.

Concepts: x x x + x x x = x x x x x x
3 + 3 = 6

Two sets of 3 are equivalent to one set of 6.
Two three's are 6.

2 x 3 = 6

Have 3 girls come to the front of the room to sit at one side of the reading table. 3 more girls come to sit on the other side of the table. Choose a child to tell the number story of the dramatization. Story expected: "Three girls were reading, then three more came. How many girls were reading?" Ask a child to write a number sentence 3 + 3=6. How may 3's did you add? What is two 3's. Write on the board, Two 3's = 6. Further develop the concept with two 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's.

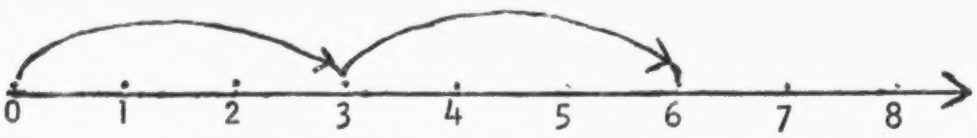
Activity:

1. Form teams in the following manner: First, have 2 students on each team, then 3, then 4, and so on. As the teams form, have students name the number of students on each team and on both teams.
2. Chain gang: One student names a number (4) and the next student gives a sum which has two equal addends using that number. The next student names a new number (6) and the next student says twice 6 is 12, and so on. The object is to get around the room without breaking the chain.

0370 Multiplication developed from the number line.

Draw a line on chalkboard. Have a student show the moves for addition of 2 equal addends, beginning at 0 and moving to the right 2 moves. Ex. 3 + 3 means to move from 0 to 3 to 6. Ask, "How many times did you move?" response, "Twice," write 2 x 3 = 6 on the board. Ask a student to explain the x symbol. Solve the equation, compare with moves on number line. Continue drill to insure that the student understands the connection between multiplication and addition of equal addends.

2 x 3 = 6
or
3 + 3 = 6



0410 Commutativity, a property of multiplication.

000 000 — 2 sets of 3 objects is 6 objects

00 00 00 — 3 sets of 2 objects is 6 objects

two 3's is 6
so 2 x 3 = 3 x 2

three 2's is 6
so 2 x 3 = 3 x 2

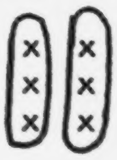
The teacher should review the commutative property of addition, (see 0150), to demonstrate that the order in which we add two numbers does not affect the end result. Will the order in which we multiply affect the result? no

To illustrate further, see 0360 - arrays.

3 columns XXX	The two rows have 3 objects each for a total of 6 objects.
2 rows xxx	The three columns have 2 objects each for a total of 6 objects.

The teacher may devise activities to illustrate other examples.

0540 Division developed from partitioning into equivalent sets.



How many sets of 3 objects can be made from a set containing 6 objects, or if six objects are separated into two equivalent sets, how large will each set be?

The students should know the relationship between addition and multiplication, multiplication facts in which 2 is the multiplier, and the commutative principle of multiplication.

Arrange 8 books (or other objects) in a row on a table. Have a child dramatize a number story such as: Mary had 8 books. She gave 4 books to each of her friends. How many books did Mary give to her friends? Have Mary take 4 books to a friend, then 4 more to another friend. Then ask, "How many friends got 4 books each?" "How many 4's in 8?" "How many sets of 4 did Mary make?" "How many 4's in 8?"

As the child works with discs, develop more statements as above for sets of 3's in 6, sets of 2's in 4, etc.

Note that we are partitioning a given set into two equivalent sets. The same approach may be applied to three equivalent sets, and others.

0570 Division, the inverse of multiplication.

If 6 x ☐ = 12 then 12 ÷ 6 = ☐

Review related multiplication facts, division facts with 2 as the divisior, and related division facts with 2 as the quotient.

Use large counting objects for an example. Write on the board how many 6's in 12? How many 2's in 12? Use dramatization to develop this. Then illustrate the relationship of division to multiplication.

2 x 7 = 14   14 ÷ 7 = 2

0330 Multi-digit subtraction with regrouping.

$$\begin{array}{r} 52 \\ \underline{-25} \\ 27 \end{array} \qquad \begin{array}{l} \text{5 tens and 2 ones} = \text{4 tens} + \text{12 ones} \\ \text{then } 40 + 12 \\ \quad \underline{-20 + \ \ 5} \\ \quad\ \ 20 + \ \ 7 = 27 \end{array}$$

As a background, use a number line at the board as children use one at their seats. Write on the board, 37 + 6 = ☐. Use number line for solution. Write inverse equation 43 - 6 = ☐. Find solution. Develop 13 - 6 = 7, 23 - 6 = 17, 33 - 6 = 27, 10 - 4 = 6, 20 - 4 = 16, 30 - 4 = 26.

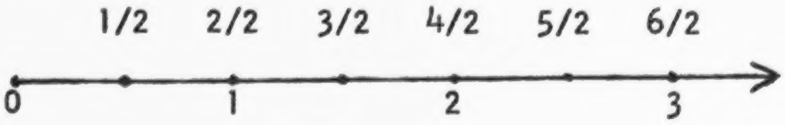
Ask the following questions: What is the sum? What are the addends? How many tens in the sum? How many tens in the missing addend? What is the basic number fact?

Have the children discover that the number of tens in the missing addend is 1 less than the number of tens in the sum. Use abacus or place value chart-rename 1 ten as 10 ones.

$$\begin{aligned} 45 - 6 &= (30 + 15) - 6 && \text{rename} \\ &= 30 + (15 - 6) && \text{regroup} \\ &= 30 + 9 \\ &= 39 \end{aligned}$$

1020 Fractions developed from distances on number line.

Use ruler to develop $\frac{1}{2}$, $\frac{1}{4}$.
Count halves 1/2, 2/2, 3/2, 4/2



2/2 is another name for $\boxed{1}$

4/2 is another name for $\boxed{2}$

6/2 is another name for $\boxed{3}$

To reinforce: Have children draw line segments using ruler. Have them label the whole number points or the line then the half way points.

1090 Ordering. Greater than; less than; equal to

Larger than, less than has been taught. By comparing sizes of sets of sticks such as 14 is greater than 12 or 7 + 8 is greater than 10, 9 is less than 15, 8 + 8 is equal to 16. To associate the symbol with the words-erase words-substitute the symbol noting that the open area of the symbol is toward the larger numeral and the point toward the smaller numeral.

$100 < 60 + 50$ $4 + 8 > 10$

$100 = 50 + 50$ $4 + 8 = 12$

$100 > 50 + 40$ $4 + 8 < 13$

$1/2 > 1/3$, $1/2 > 1/4$, $1/4 < 1/3$, $1/4 < 1/2$

3100 Rounding numbers - estimating answers.

Review sequence charts. Note numberals that are nearest another numeral.

If you were on an elevator at the 27th floor level. Would it be quicker to go to the 30th floor or the 20th floor? Repeat this drill for several examples developing the concept of rounding or estimating numerals.

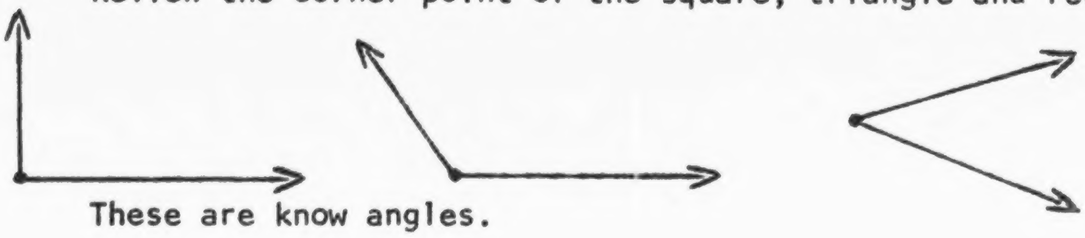
37 rounds to nearest 10 as 40
86 rounds to nearest 10 as 90

Flash cards may be made by each child for 10 to 100. Arrange these cards on each desk so all are in view. Write numerals on chalkboard, ask children to flash the card with the nearest number.

5190 Angles

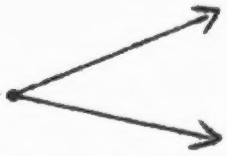
Concept to be developed: An angle is made up of two rays with a common end point.

Review the corner point of the square, triangle and rectangle.

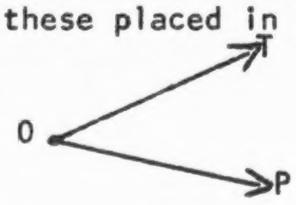


These are know angles.

Procedure. ⟵ Have charts showing these placed in front of room.



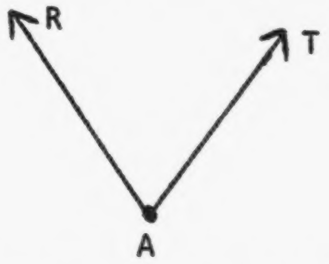
This is an angle.
It is made up of two rays with a common end point called a corner point.

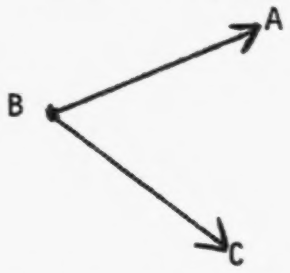


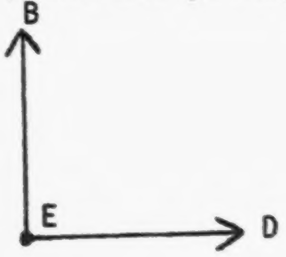
This angle is made up of the two rays.

$\overrightarrow{OT}$ and $\overrightarrow{OP}$ We may name this angle as $\angle$ TOP or as $\angle$ POT.

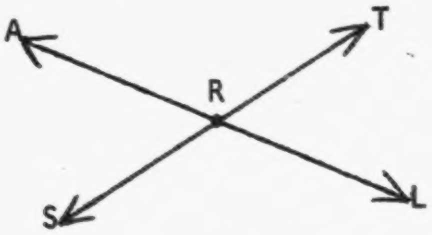
Draw various angles, labeling the rays. Emphasize the end point and names for the angles. (note: rays have only one end point)







Note that a number of lines may be drawn through one point.



$\overleftrightarrow{AL}$ and $\overleftrightarrow{ST}$ are lines. They meet at the point R. $\overrightarrow{RA}$ is a ray. Have children name 3 other rays from R. Then have the angles named. ($\angle$ T R L is an angle)

Enrichment activity:

Children may draw rays meeting at a common end point and label as they like. They may list the angles and rays.

VOCABULARY - GRADE 3

1. braces
2. cardinal number
3. the empty set
4. equivalent
5. numeral
6. greater than
7. less than
8. ones' place
9. tens' place
10. hundreds' place
11. one-to-one matching
12. place value
13. set
14. subset
15. total value
16. addend
17. sum
18. comparing
19. difference
20. remainder
21. equation
22. Family of facts
23. missing addend
24. opposites
25. remaining
26. separating
27. angle
28. area
29. center
30. circle
31. curve
32. closed curve
33. diagonal
34. diameter
35. end point
36. foot
37. half inch
38. quarter inch
39. inch
40. length
41. line
42. line segment
43. measurement
44. parallel
45. path
46. plane
47. polygons
48. quadrilateral
49. radius
50. ray

51. rectangle
52. region
53. triangle
54. set of points
55. square
56. square inch
57. yard
58. vertex
59. surface
60. standard unit
61. simple closed curve
62. digit
63. inverse operations
64. joining
65. division
66. equal addends
67. factor
68. joining equivalent sets
69. multiplication
70. product
71. times
72. expanded numeral
73. adding equal addends
74. common factor
75. matching sets
76. base ten
77. common multiple
78. even number
79. odd number
80. periods
81. placeholder
82. prime number
83. finite
84. inifinite
85. million
86. multiple
87. quotient
88. cube
89. cylinder
90. graph
91. lattice
92. parallelogram
93. symmetry
94. space
95. sphere
96. volume
97. associative property
98. closure property
99. commutative property
100. distributive property

101. identity element

102. temperature

103. degrees

104. weight

MATERIALS - GRADE 3

1. folding flannel board
2. felt cutouts of shapes, numerals, and symbols
3. individual number line
4. colored counting cubes
5. abacus
6. visual fractions
7. counting slide rule
8. ABC of solids
9. tactile time teacher
10. individual student abacus
11. tactile numeral board
12. minute minder
13. place value chart
14. hundreds' chart
15. colored rods
16. squared paper
17. yardstick
18. individual rulers
19. geometric shapes
20. sticks for bundles of tens and ones
21. flash cards for +, -, x, ÷
22. pint bottles
23. quart bottles
24. gallon jug
25. individual clocks

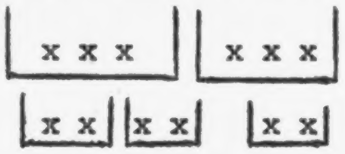
IMPORTANT ITEMS

Item	Description	Grade 3		
		B	M	E
0190	Elementary facts of addition.	M		
0200	Multi-digit addition-no regrouping.		M	
0210	Addition-two addends-regrouping.		M	
0250	Subtraction-developed from subsets.			M
0360	Multiplication-developed from arrays.		I	
0470	Multi-digit multiplication no regrouping.		I	
0480 *	Multi-digit multiplication regrouping.			I
0490 *	Multiplication more than two factors.			I
0510 *	Multipler or powers of ten as factors.		I	
0575	Distributivity of division over addition.			I
0610	Elementary facts of division.		I	
0620 *	Divisor less than ten-dividend not regrouped.			I
1130	Addition of fractions-developed from number line.		I	
1190	Common fraction notation-equal denominators.		I	
3020 *	Equivalent common fractions.		I	
3090 *	Use of commas to separate periods.		I	
7080 *	Number patterns, rules for divisibility.			I
7090 *	Arithmetic progressions.		I	
0035	Cardinal numbers beyond 10.		M	
0070	Ordinal counting-sequence.		M	
0090 *	Backward-rote counting.			M
0120	Addition-union of disjoint sets.			M
0130	Addition-developed from number line.			M
0260	Subtraction-developed from number line.			M
0450 *	Multiplication-property of zero.		I	

* Difficult Items

0450 Property of zero in multiplication

The teacher may review the commutative property of multiplication for utilization with this topic.



2 threes are 6

3 twos are 6

or 2 x 3 = 3 x 2

The teacher may begin with 2 boxes such as this.

How many objects are in each box? If the 2 boxes are joined, how many objects will there be? Therefore 2 x 0 = 0
Can you write the commutative form of 2 x 0? The child should respond with "the commutative for of 2 x 0 is 0 x 2". What is 0 x 2? Illustrate with other similar examples and with this summarizing equation. 2 x 0 = 0 x 2 = 0

It is rather difficult to use a physical world example for 0 x 2 which means 0 twos, hence the commutative property review may avoid the difficulty.

0090 Other counting, backward, rote, etc.

102, 101, 100, __________,__________

198, 196,______, ______, 190, ______, ______

90, 91, 92, ______, ______, ______, 96, ______, ______, ______

Review rote counting by use of the number line, pennies and hundreds' chart. In skip counting remove or include progressively as many objects as needed. Children count as they include or remove objects. Since developing the ability to discover patterns is vital to mathematics, let the children investigate a wide variety of patterns on the numeral chart by covering or circling the missing numerals.

7090 Arithmetic progressions

Arithmetic progression is a sequence of numbers each differing from the preceding number by a fixed amount.

3, 6, 9, 12 the constant difference is 3

18, 16, 14, 12 the constant difference is 2

To explain example 1, begin by using the number line and sliding three units on the number line. Follow up by using objects on the flannel board and having children add three progressively. The children may also use geometric shpes such as squares or rectangles at their seat.

If the children discover patterns that aid them, they are alert and should be encouraged. For example adding 9 is the same as adding (10 - 1), or adding 10 and subtracting 1, the tens digit increases from one sum to the next, and the ones' digit decreases.

6, 11, 16, 21, 26, 31 See pattern in ones' place when five is added progressively.

7080 Rules for divisibility

Is 24 evenly divisible by 2? All even numbers can be divided exactly by 2.

It is important that students know what even numbers are. An even number has 2 as a factor. Zero is an even number because it has 2 as one of its factors 2 x 0 = 0. All numbers with 0, 2, 4, 6, 8 in the one's place are even numbers. When you count by 2's starting at 0, the numbers you name are even numbers.

When you add any number to itself, the answer is an even number. 3 + 3 = 6 or 2 sets of threes is 6. Two is a factor of 6.

All numbers represented by numerals ending in 0 or 5 are exactly divisible by five.

How many nickels in 15 cents? Three! How many nickels in 40 cents? Eight! Illustrated that 3 fives = 15 and 8 fives = 40 with the child noticing the numerals 0 and 5 in the unit's or one's place.

If the child has difficulties with money other examples may be used.

3090 Using commas to separate into periods.

By this time the students are familiar that 9 ones and one more one is 1 ten; 9 tens and one more ten is 1 hundred; 9 hundreds and one more hundred is 1 thousand. This may be reviewed by using the abscus, the place value box or chart, and/or the number line.

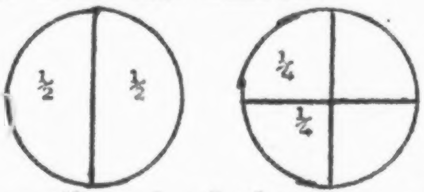
The thousand place is fourth from the right. You see that in each numeral a comma is used between the three figures at the right and those that stand for thousands.

Thousands' Period			Ones' Period			
H	T	O	H	T	O	
	1	6	7	2	6	= 16, 726
3	8	2	4	9	3	=382, 493
2	0	4	9	0	6	=204, 906

H means hundreds
T means tens
O means ones

3020 The equivalent common fraction notation.

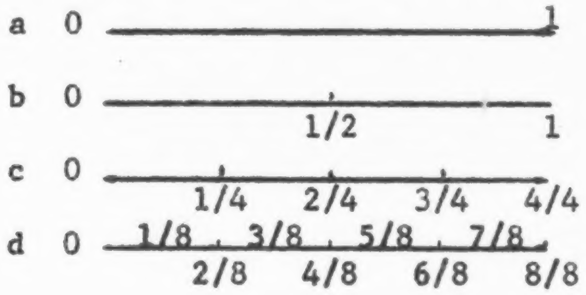
Place a circular felt cut-out on the flannel board. Cover the region with 2 parts of the same size. Ask what part of the region is being covered by each. Then cover the region with 4 parts of the same size. Ask what part is being covered by each. Remove 2 of the 4 parts and ask what fraction tells how many parts were removed (2/4). Then remove the half region and ask the students if the two fourth-regions match the one half-region. Write the numerals ½ and 2/4 on the board and emphasize that both name the same number.



2/4 = ½

Several pieces of cardboard of the same size may be used for matching and comparing. Then match the regions 4 eighths with ½, 2 eighths with ¼, etc.

The number line is also a good device to develop this concept.



0620 Divisor less than ten, dividend not regrouped, no remainder

8 ÷ 2 = 4 ; 9 ÷ 3 = 3

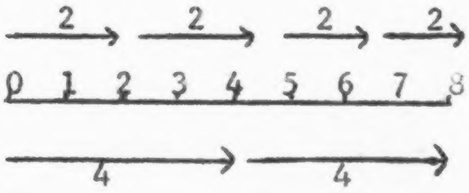
By use of the flannel board or concrete objects review 2 fours = 8, 4 twos = 8, 2 x 4 = 8, and 4 x 2 = 8. If two 4's equals 8, how many 4's are in 8? If four 2's equals 8, how many 2's are there in 8?

(0 0 0 0) a. 2 x 4 = 8 b. How many 4's are there in 8?

(0 0 0 0) c. 4 x 2 = 8 d. How many 2's are there in 8?

a. 2 x 4 = 8 b. 4 x 2 = 8 c. 8 ÷ 4 = 2 d. 8 ÷ 2 = 4

The sign ÷ is the division sign. You read the division sentence this way: Eight divided by four is 2. It means that the number of 4's in 8 is 2. This concept may be further developed on the number line.



0510 Powers or multiples of ten as factors.

Begin by reviewing 4 x 2 = 8; 4 x 2 ones = 8 ones. By using bundles of sticks or cards, extend this concept to 4 x 20 or 4 x 2 tens = 8 tens or 80. Emphasize the relationship between multiplying ones and multiplying tens.

Exercises such as the following may be written on the chalkboard and illustrated by single sticks and bundles of ten sticks.

3 x 3 = 9   3 x 30 = 90   4 x 5 = 20   4 x 50 = 200

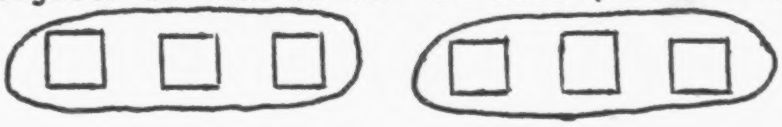
The child will also learn that the associative property is useful in justifying multiplication involving multiples of 10. He will learn to approach the problem 3 x 30 =   in terms of the base of his system of numeration.

$$\begin{aligned} 3 \times 30 &= 3 \times (3 \times 10) \\ &= (3 \times 3) \times 10 \\ &= 9 \times 10 \\ &= 90 \end{aligned}$$

0490 Multiplication with more than two factors without regrouping.

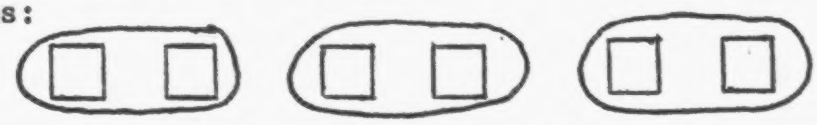
2 x 1 x 3 = □

Since the operation of multiplication has the associative property, there are two ways to represent the above product. Consider: 2 x (1 x 3) =   . The 2 represents the number of sets, and the (1 x 3) represents the number of objects in each of the two sets (one set of three). Using set diagrams:



Two sets of (one set of three). The product 2 x (1 x 3) represents the total number of objects. Thus 2 x (1 x 3) = 6. Now consider: (2 x 1) x 3 =   .

By the commutative property of multiplication, this can be written 3 x (2 x 1) =   . The 3 represents the number of sets, and the (2 x 1) represents the number of objects in each set (one set of two). Using set diagrams:



Three sets of (one set of two)

The product 3 x (2 x 1) represents the total number of objects. Therefore, 3 x (2 x 1) = 6 or using the commutative property of multiplication, (2 x 1) x 3 = 6.

0480 Multi-digit multiplication of two factors with regrouping.

$$\begin{array}{r} 87 \\ \underline{\times 3} \\ 21 \\ \underline{240} \\ 261 \end{array}$$

Recall the distributive principle by writing the above in sentence form.

$$\begin{aligned} 3 \times 87 &= (3 \times 80) + (3 \times 7) \\ &= 240 + 21 = \boxed{261} \end{aligned}$$

Let the pupils tell what (3 x 80) and (3 x 7) are and let them add the products.

Now write the same example in vertical form as shown in example 1. Get agreement to write the product of the ones first in examples in vertical form. Show that it is easy to multiply first with the figure just above the multiplier and then with the figure in tens' place. Let the pupils discover that the example is like the sentence, except for the form.

INTERMEDIATE
SECTION

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0010-0100	BASIC CONCEPTS OF WHOLE NUMBERS.											
0100	Definition: Set of whole numbers.	1			M							
0080	Skip counting.	1			M							
0100	Ordering-equal, greater, less.	K					M					
0110-0200	PROPERTIES OF ADDITION.											
0110	Properties of addition.		I					M				
0140	Closure.		I					M				
0150	Commutativity.	K			M							
0160	Associativity.	1				M						
0170	Zero-the identity element.	1				M						
0230	More than two addends-regrouping.	2			M							
0240-0330	PROPERTIES OF SUBTRACTION-WHOLE NUMBER.											
0240	Definition-binary operation.			I			M					
0270	Inverse of addition.	1			M							
0280	Property of zero.	1				M						
0290	Non-closure, non-commutative, non-associative.		I			M						
0310	Elementary facts of subtraction.	1	M									
0320	Multi-digit subtraction-no regrouping.	1	M									

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0330	Multi-digit subtraction-regrouping.	2			M							
0340-0520	PROPERTIES OF MULTIPLICATION-WHOLE NUMBER.											
0340	Definition-binary operation.		I					M				
0350	Developed from union of equivalent set.	2	M									
0360	Developed from arrays.	3				M						
0370	Developed from number line.	2			M							
0380	Developed as repeated addition.	1			M							
0390	Developed from product sets.					I						X
0400	Closure.							I				X
0410	Commutativity.	2				M						
0420	Associativity.	3							M			
0430	Distributivity of multiplication over addition.	3							M			
0440	One, the identity element.	3			M							
0450	Property of zero.	3			M							
0460	Elementary facts of multiplication.	1			M							
0470	Multi-digit multiplication, two factors-no regrouping.	3			M							
0480	Multi-digit multiplication, two factors-regrouping.	3				M						
0490	More than two factors-no regrouping.	3			M							

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0500	More than two factors-regrouping.				I		M					
0510	Multiples or powers of ten as factors.	3					M					
0520	Numbers in exponential form.						I					X
0530-0670	PROPERTIES OF DIVISION WHOLE NUMBERS.											
0530	Definition-binary operation.						I					X
0540	Developed from partitioning-equivalent set.	2			M							
0550	Developed as successive subtraction	3					M					
0560	Developed from the number line.	3					M					
0570	Inverse of multiplication.	2					M					
0575	Distributivity of division.	3									M	
0580	Property of one.	3					M					
0590	Zero not a divisor.						I					I
0600	Nonclosure, noncommutative, nonassociative.				I							X
0610	Elementary facts of division. Divisor less than 10.	3			M							
0620	No regrouping - no remainder. Divisor less than 10.	3			M							
0630	Dividend not regrouped - remainder. Divisor less than 10.	3				M						
0640	Dividened regrouped - no remainder. Divisor less than 10.	3					M					
0650	Dividend regrouped - remainder.				I		M					

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
0660	Divisor ten or greater.				I					M		
0665	Powers or multiples of 10 as divisors.	3							M			
0670	Numbers expressed in exponential form.								I			I
0680	Solve problems - two operations.	3					M					
0690	Problems - more than two operations.				I							X
0700	Raise numbers to powers - find roots.							I				X
1000-1100	BASIC NUMBER CONCEPTS WITH FRACTIONS.											
1000	Definition - set of nonnegative fractions.						I					X
1010	Developed from subset of a given set.	1					M					
1020	Developed from distances on number line.	2								M		
1030	Developed from plane and solid regions.	1								M		
1040	Whole number related to set of fractions.				I						M	
1050	Renaming from set of fractions.	3								M		
1060	Definition: equality.			I								I
1070	Expressed as terminal or repeating decimal.							I				X
1080	Counting.	3					M					
1090	Ordering: greater, less, equal to.	2								M		
1100	Density.				I							X
1110-1210	PROPERTIES OF ADDITION OF FRACTIONAL NUMBERS.											
1110	Definition: binary operation.						I				M	
1120	Developed from union of disjoint sets.			I			M					
1130	Developed from the number line.	3					M					
1140	Developed from plane or solid regions.	2					M					

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
1150	Closure.					I						X
1160	Commutativity.	3						M				
1170	Associativity.				I						M	
1180	Zero, the identity element.	3					M					
1190	Common fraction notation, equal denominator.	3					M					
1200	Common fraction unequal denominators.						I				M	
1210	Decimal fraction notation.							I			M	
1220-1310	PROPERTIES OF SUBTRACTION WITH FRACTION.											
1220	Definition: binary operation.				I			M				
1230	Developed in relation to subsets.				I					M		
1240	Developed from the number line.				I			M				
1250	Developed from plane or solid regions.	3					M					
1260	Inverse of adding fractions.	3						M				
1270	Property of zero.	3						M				
1280	Non-closure, non-commutative, non-associative.						I					X
1290	Common fractions, equal denominators.	3					M					
1300	Common fractions, unequal denominators.						I				M	
1310	Decimal fraction notation.							I			M	

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
1320-1440	PROPERTIES OF MULTIPLICATION WITH FRACTIONS.											
1320	Definition: binary operation.							I				X
1330	Developed from addition of equal fractions.							I			M	
1340	Developed from arrays.							I				X
1345	Developed from the number line.							I				X
1350	Developed from plane and solid regions.							I				X
1360	Closure.							I				X
1370	Commutativity.							I			M	
1380	Associativity.							I				X
1390	Distributivity of multiplication over addition.									I		X
1400	One, the identity element.							I			M	
1410	Property of zero.							I			M	
1420	Multiplicative inverse (reciprocal).							I			M	
1430	Common fraction notation.							I			M	
1440	Decimal fraction notation.							I				X
1450-1550	PROPERTIES OF DIVISION WITH FRACTIONAL NUMBERS.											
1450	Definition: binary operation.									I		X
1460	Developed from successive subtraction.							I		M		

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
1470	Developed from the number line.							I				X
1480	Developed from plane and solid regions.							I				X
1490	Inverse of multiplying by a fraction.							I			M	
1500	Closure									I		X
1510	Property of one.							I			M	
1520	Zero not a divisor.									I		X
1530	Noncommutative, nonassociative.									I		X
1540	Common fraction notation.							I				X
1550	Decimal fraction notation.							I				X
1560	Problems with two or more operations.						I					X
2000-2040	BASIC NUMBER CONCEPTS WITH INTEGERS.											
2000	Definition: set of integers.									I		X
2010	Developed from the number line.									I		X
2030	Ordering: greater, less, equal to.						I					X
2040	Directed numbers; absolute value.									I		X
2050-2100	PROPERTIES OF ADDITION WITH INTEGERS.											
2050	Definition: binary operation.										I	X
2053	Developed from number line.										I	X
2055	Developed from physical world situations.										I	X

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
2060	Closure.										I	X
2070	Commutativity.										I	X
2090	Zero, the identity element.										I	X
2100	Additive inverse.										I	X
2110	Algorithm for addition of integers.										I	X
2120-2170	PROPERTIES OF SUBTRACTION-INTEGERS.											
2123	Developed from number line.										I	X
2125	Developed from physical world situations.										I	X
2170	Algorithm for subtraction of integers.										I	X
3000-3160	NUMERATION AND NOTATION.											
3000	Difference between number and numeral.	1		M								
3010-3040	DIFFERENT NUMERALS FOR SAME NUMBER.											
3015	Expanded notation for fractions.	3					M					
3020	Equivalent common fractions.	3								M		
3025	Equivalent mixed numerals.		I							M		
3030	Equivalent decimal fractions.							I				X
3035	Equivalent percent notation.									I		X
3040	Other names for a number.	1				M						

Item	Description	Pre Introduction	Fourth				Fifth			Sixth			Mastery Beyond Elem.
			B	M	E		B	M	E	B	M	E	
3050-3110	ABILITY TO READ AND WRITE.												
3080	Beyond tens (one hundred billion).	1							M				
3090	Use commas to separate periods.	3					M						
3100	Ability to round numbers for estimates.	2										M	
3110	Decimal fractions to ten thousandths.								I			M	
3120	Use exponential notation.								I				X
3140	Knowledge of historical development.						I						X
3150	Historical systems of numeration.	2											X
3160	Place value related to decimal system.							I					X
4000-4125	BASIC CONCEPTS OF SETS.												
4000	Meaning: set and members (elements).	K					M						
4050	One-to-man correspondence.	1			M								
4060	Subsets.	K											X
4070	Empty.	1			M								
4080	Intersecting.					I							X
4090	Disjoint.					I							X
4100	Finite.				I							M	
4110	Infinite.				I							M	
4120	Universal and complement.				I								X

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
4125	Solution set.		I									X
4130-4170	FORMING NEW SETS BY.											
4130	Union.						I					X
4140	Intersection (venn diagrams).						I					X
4150	Subsets.	3									M	
4160	Product sets.									I		X
4170	Partitioning sets.								I			X
5010-5080	RECOGNIZE GEOMETRIC FIGURES AND IDEAS.											
5010	Geometric figures in environment.	K										X
5020	Geometric designs and patterns.	K										X
5030	Relations without measure (size, position).	K										X
5040	Two-dimensional figures (plane).	K						M				
5050	Three-dimensional figures (solid).	2						M				
5060	Curves; simple, closed, open.	K						M				
5070	Regions: interior, exterior.	1			M							
5080	Representations: point, line, plane.	K				M						
5090-5200	DESCRIBE AND ANALYZE GEOMETRIC FIGURES.											
5090	Point.	2										X
5100	Line, line segments, rays.	2										X

Item	Description	Pre Introduction	Fourth B	Fourth M	Fourth E	Fifth B	Fifth M	Fifth E	Sixth B	Sixth M	Sixth E	Mastery Beyond Elem.
5110	Planes.					I						X
5120	Space.					I						X
5130	Curves, simple, closed, open.	3										X
5140	Regions interior exterior.					I						X
5145	Polygons.					I						X
5150	Triangles.					I						X
5160	Quadrilaterals.					I						X
5170	Other polygons.					I						X
5180	Circles.	1										X
5190	Angles.	2										X
5200	Three-dimensional figures.							I				X
5210-5230	CONSTRUCT OR DRAW GEOMETRIC FIGURES.											
5210	Line constructions (segments, rays).	3									M	
5220	Two-dimensional figures.					I						X
5230	Three-dimensional figures.										I	X
5234-5300	ABILITY TO MEASURE GEOMETRIC FIGURES.											
5234	Compare sizes, shapes, distances.					I						X
5240	Congruency.										I	X
5250	Similarity.					I						X

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
5255	Scale drawing.							I				X
5260	Line segments (ruler, compass, other).	1									M	
5270	Angles with protractor or compass.							I				X
5280	Finding perimeter or circumference.						I					X
5290	Area of plane regions.						I					X
5300	Volume of solid regions.						I					X
6000-6030	MEASUREMENT-NONGEOMETRIC QUANTITIES.											
6000	Meaning of measure.	2										X
6005	Rule for greatest common error.								I			X
6010	Development of standard units.								I			X
6020	Development of nonstandard units.	3										X
6030	English and metric units.	1								M		
6040-6120	ABILITY TO APPLY UNITS TO MEASURE.											
6040	Money.	1					M					
6050	Time.	1										X
6060	Distance.	3										X
6070	Liquids.	1										X
6080	Temperature.	1										X
6090	Weight.	3										X

Item	Description	Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
6100	Dry measure.			I							M	
6110	Quantity (dozen, gross).	1									M	
6120	Denominate numbers.		I									X
7000-7080	NUMBER PATTERNS: ELEMENTARY NUMBER THEORY.											
7000	Odd-even numbers.	1									M	
7010	Factors and primes.			I								X
7020	Composites: perfect numbers.							I				X
7040	Composites: general.							I				X
7050	Greatest common factor.					I						X
7060	Least common multiple.						I					X
7070	Understand unique factorization.			I								X
7080	Rules for divisibility.	3										X
7090-7150	ABILITY TO USE NUMBER SEQUENCES.											
7090	Arithmetic progressions.	3									M	
7100	Geometric progressions.							I				X
7110	Triangular numbers.							I			M	
7120	Square numbers.							I			M	
7140	Grid games.	3										X
7150	Fibonacci numbers.										I	X

Item	Description	Pre Introduction	Fourth			Fifth			Sixth			Mastery Beyond Elem.
			B	M	E	B	M	E	B	M	E	
7160	Ability to use short cuts.	3										X
8000-8150	OTHER TOPICS.											
8000	Concepts of ratio and proportion.							I				I
8010	Basic concepts of percent.							I				X
8020	Graphing-number line.	3										X
8030	Graphing-coordinate plane.									I		X
8040	Ordered pairs-coordinate plane.									I		X
8050	Tables, charts, graphs.	3										X
8060	Average, mean, median.				I							X
8080	Probability.										I	X
8090	Clock arithmetic.								I			X
8100	Modular arithmetic.								I			X
8110	Without numbers.					I						X
8130	Logic.										I	X
8140	Functions and relations.				I							X
8150	Estimate and verify answers.	3										X
8160	Relations (properties of).	1										X

VOCABULARY - GRADE 4

1. addend
2. equation
3. inequality
4. renaming
5. sets
6. subsets
7. area
8. angle
9. congruent
10. diameter
11. parallel
12. quadrilateral
13. radius
14. square inch
15. triangle
16. cross product
17. equivalent
18. product
19. cardinal number
20. denominator
21. numerator
22. region
23. subregion
24. cardinal region
25. commutative property
26. associative property
27. distributive property
28. factor
29. exponent
30. intersection
31. prime number
32. common factor
33. quotient
34. product
35. contradict
36. intersection
37. parallel
38. plane
39. pyramid
40. space
41. vertex
42. sphere
43. unit
44. volume
45. functions
46. removing subsets
47. expanded numerals
48. metric systems
49. universal sets
50. array

51. finite

52. infinite

53. identity element

MATERIALS • GRADE 4

1. flannel board (with stand)
2. felt shapes
3. objects to put conveniently in bundles of ten - tongue depressors
4. abacus (large one)
5. plastic shapes
6. numerals and symbols
7. number line
8. colored rods
9. place value chart
10. grid blackboard
11. aluminum liquid measures
12. minute minder
13. mathematical balance
14. visual fractions
15. flash cards
16. pipe cleaners - large
17. geometric figures
18. graph paper
19. string
20. oak tag
21. rulers

IMPORTANT ITEMS

Item	Description	Grade 4		
		B	M	E
0310	Elementary facts of subtraction.	M		
0330 *	Multi-digit subtraction-regrouping.			M
0230 *	More than two addends-regrouping.			M
0460 *	Elementary facts of multiplication.			M
0370	Subtraction-developed from number line.			M
0270	Subtraction-inverse of addition.			M
0380	Multiplication developed as repeated addition.			M
0660 *	Divisor ten or greater.			I
1040 *	Whole numbers related to set of fractions.	I		
0350	Multiplication developed from equivalent sets.	M		
0650 *	Divisor less than 10, regrouping, remainder.			I
0320	Multi-digit subtraction-no regrouping.	M		
0620 *	Divisor less than 10, no regrouping or remainder.			M
0150	Commutativity-addition.			M
0140	Closure-addition.	I		
0540	Division-developed from equivalent sets.			M
0490 *	More than two factors-no regrouping.			M
8140	Functions and relations.			I
0600 *	Division-nonclosure, non-commutative, non-associative.			I
0610	Elementary facts of division.			M
8060 *	Average, mean, median.			I
0010	Definition: set of whole numbers.			M
0240	Definition-subtraction-binary operation.		I	
0290	Subtraction-nonclosure, non-commutative, non-associative.	I		

* Difficult Items

METHODOLOGY - GRADE 4

0600 Nonclosure, noncommutativity, nonassociativity. Division of whole numbers.

$5 \div 2 \neq$ whole no.

$12 \div 4 \neq 4 \div 12$ $\qquad$ $4 \times 2 = 2 \times 4$

$(24 \div 4) \div 2 \neq 24 \div (4 \div 2)$

The student should be aware of basic number sense in reference to the number line and number sentences, closure, commutativity, and associativity of multiplication. Continue building number sentences by using the left to right process in reading all number sentences. Establish the fact that the first number is divided by the second on both sides of the symbol of comparison.

Suggest mental exercise game for multiplication facts using the game of baseball as motivation. Suggest a number and ask for factors that will produce the given number. This can be revised to suit the teacher's needs.

$\square \times 2 = 5 \quad (2 \times 2 = 4), \ (3 \times 2 = 6)$

No whole number times 2 will equal 5

$5 \div 2 \neq$ a whole number

$12 \div 3 \neq 3 \div 12$

$4 \neq \frac{1}{4}$

$4 \times 3 = 12; \ 4 \times 12 \neq 3$

$(24 \div 4) \div 2 \neq 24 \div (4 \div 2)$

$6 \div 2 \neq 24 \div 2$

$3 \neq 12$

0620 Divisor less than ten, dividend not regrouped, remainder.

Review basic facts of multiplication. Use the number line to show division with remainders. Review number sentences of inequalities.

$\square$ times $2 < 9$	$\square$ times $7 < 69$	$59 \div 6 = \square$
$4 \times 2 = 8$ $5 \times 2 = 10$	$8 \times 7 = 56$ $9 \times 7 = 63$ $10 \times 7 = 70$	What is the largest number times 6 that is less than 59?

Show children why it is best to use the largest number possible. Make sure it is understood that the remainder must be less than the divisor.

__8060__ Measures of central tendency - average.

Place on a desk 3 different sets of books (perhaps 4 - 5 - 3). Have a student arrange the books so that all 3 sets have the same number of books. This indicates the basic notion of average. Now have the children list the number of books from each set 4, 5, or 3; then find the total. If we divide the total number of books by the number of sets, 12 ÷ 3, we have the average number of books for each of the three sets.

__0660__ Divisor ten or greater.

```
        21
      ┌──────┬───────────────┬───────┐
      │      │               │ check │
   21 │ 252  │               │       │
        240  │  12 x 20 =    │ 240   │
      ───────┼───────────────┤       │
         12  │               │       │
         12  │               │       │
        ─────┤  12 x  1 =    │  12   │
          0  │        21     │ 252   │
                             └───────┘
```

Review multiplication facts 1 through 12, place values such as ones, tens, and hundreds, and basic division facts using divisors less than ten.

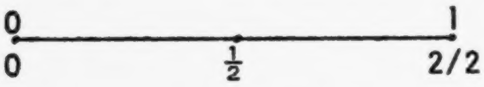
Division is a form of successive subtraction. Using divisors equal to or greater than 10 can be related to division using divisors less than 10. Rename the dividend in terms of tens and ones.

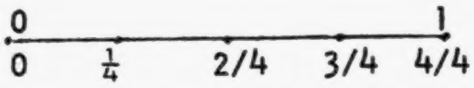
Activities to motivate learning might include a game of football in which yardage is gained by naming the correct quotients using divisors less than greater than or equal to 10.

	Goal
10	_____121 ÷ 11_______
20	_____144 ÷ 12_______
30	_____70 ÷ 10_______
40	_____66 ÷ 6_______
50	_____50 ÷ 10_______
40	_____24 ÷ 8_______
30	_____100 ÷ 10_______
20	_____48 ÷ 12_______
10	_____10 ÷ 2_______
	Goal

1040 Whole numbers as related to set of nonnegative rationals (fractions).

2/2 = 1   3/3 = 1   4/4 = 1   16/16 = 1   73/73 = 1   28/28 =1



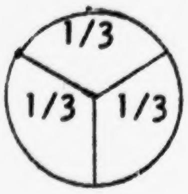


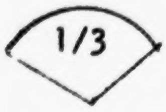
Use the number line to review number concepts. Discuss division of one pie etc. to be divided equally. Establish need for dividing numbers into smaller parts.

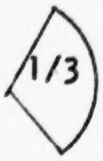
The increased number of pieces does not increase the basic concept of the size of the original piece. Establish the fact that the numerator shows the number of pieces while the donominator shows the number of divisions in the whole object. Show how to divide the bottom number into the top to determine if the number is equal to one or more. 4/4 = 1, 8/4 = 2,

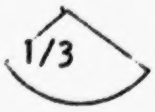
$$\begin{array}{r} 1 \\ 4\overline{)4} \\ 4 \end{array} \qquad \begin{array}{r} 2 \\ 4\overline{)8} \\ 8 \end{array}$$

Establish puzzle games to show how many odd shaped figures can form a new figure.









0490 Multiplication with more than two factors withour regrouping-whole numbers.

$(2 \times 3) \times 4 = 24$

Review basic multiplication facts. The student should be able to formulate a new group as the end result of multiplication using two numbers.

Develop mental process of finding a primary product as the first step in developing the final product. Beginning process should be kept as simple as possible at this level. Extent concepts to developing the sense for finding volumns at later levels.

$(6 \times 5) \times 8 = 30 \times 8 = 240$ $7 \times 4 \times 9 = 28 \times 9 = 252$

Multiply first two factors, then multiply that product by the next factor. This process can be expanded to four or more factors.

$3 \times 6 \times 2 \times 5 = 18 \times 2 \times 5 = 36 \times 5 = 180$

0330 Multi digit subtraction with regrouping.

Review of the basic subtraction facts. Two basic ideas involved in renaming are: every number has many names, addition is associative.

Demonstrate subtraction on the abacus or place value chart. As you do, write the example on the board using expanded numerals.

$$\begin{array}{r} 632 \\ 187 \end{array} \qquad \begin{array}{r} 600 + 30 + 2 \\ -\underline{(100 + 80 + 7)} \end{array}$$

Since you cannot subtract 7 ones from 2 ones rename the number as:

$$\begin{array}{r} 600 + 20 + 12 \\ \underline{(100 + 80 + 7)} \end{array}$$

Use the same concept with the tens' and hundreds' place values.

$$\begin{array}{r} 500 + 120 + 12 \\ -\underline{(100 + 80 + 7} \end{array}$$

The above idea is used as an introduction. Later the short way may be introduced $\begin{array}{r} 632 \\ -\underline{187} \end{array}$

0460 The ability to use the elementary facts of multiplication.

Rediscover facts of addition and multiplication by using the number line. Review multiplication facts by using addition. $5 + 5 + 5 = 3 \times 5 = 15$. Review commutative principle of multiplication and the inverse relationship of division.

$$2 \times 3 = 3 \times 2 = 6 \qquad 5 \times 7 = 35;\ 35 \div 7 = 5; \qquad 35 \div 5 = 7$$

The distributive principle is introduced as another method of discovery.

$$\begin{array}{r} 6(3 + 5) = (6 \times 3) + (6 \times 5) \\ 6\ \ (8) = 18 + 30 \qquad\qquad \\ 48 = 48 \qquad\qquad \end{array}$$

The associative property of multiplication.

$$\begin{array}{r} (2 \times 4) \times 7 = 2\times\ (4 \times 7) \\ 8 \times 7 = 2 \times 28 \qquad \\ 56\ = 56 \qquad\quad \end{array}$$

Multiplication used in division.

$$4\overline{)8} = 2 \qquad \begin{array}{l} \text{check} \\ 2 \times 4 = 8 \end{array}$$

0230 Addition of more than two addends with regrouping.

Review basic addition of facts.

$$4 + 3 + 7 = 7 + 7 = 14$$

$$5 + 9 + 6 = 14 + 6 = 20$$

$$14 + 7 = 21$$

$$19 + 6 = 25$$

$$\begin{array}{r} 43 \\ 66 \\ 79 \\ 25 \\ +\underline{16} \end{array}$$

Children at first add: 3 + 6 is 9, + 9 is 18, + 5 is 23, + 6 is 29. Later through many experiences children will learn 3, 9, 18, 23, 29. Children are challenged by doing these easy ones mentally. They enjoy playing games with this work. After this has been developed, regrouping is not too difficult. 29 = 2 tens + 9 ones; the 9 is placed in the one's place in the sum, and the 2 is added to the ten's place.

Review the basic facts of multiplication and subtraction. Use the number line or sets to establish the idea of a number of objects remaining after grouping.

Concepts of division developed from problems using two different operations:

$$44 = (7 \times 6) + 2; \qquad 38 = (8 \times 4) + 6$$

$$\begin{array}{r} 6 \\ 7\,\overline{)44} \\ \underline{42} \\ 2 \end{array} \qquad\qquad \begin{array}{r} 4 \\ 8\,\overline{)38} \\ \underline{32} \\ 6 \end{array}$$

Successive subtractions used in naming the quotient.

$$\begin{array}{rr|l} & \underline{17\ \text{R}3} & \\ 6\big) & 105 & \\ & 36 & 6 \times 6 \\ \hline & 69 & \\ & 66 & 11 \times 6 \\ \hline & 3 & 17 \end{array} \qquad\qquad \begin{array}{rr|l} & \underline{17\ \text{R}3} & \\ 6\big) & 105 & \\ & 60 & 10 \times 6 \\ \hline & 45 & \\ & 42 & 7 \times 6 \\ \hline & 3 & 17 \end{array}$$

$$105 \div 6 = 17 \text{ R}3$$

VOCABULARY - GRADE 5

1. base
2. base five
3. decimal system
4. element
5. empty set
6. equation
7. inequality
8. equivalent sets
9. expanded numeral
10. exponent
11. finite set
12. infinite set
13. greater than
14. less than
15. intersection
16. pairing
17. place value
18. power
19. set
20. subset
21. superset
22. union
23. universal set
24. separating sets
25. functions
26. open sentences
27. opposite operation
28. closed curve
29. closed surface
30. cone
31. congruent
32. cubic unit
33. diagonal
34. diameter
35. ellipse
36. hexagon
37. line segment
38. parallel
39. parallelogram
40. pentagon
41. perimeter
42. perpendicular
43. polygon
44. protractor
45. pyramid
46. quadrilateral
47. radius
48. ray
49. rectangular prism
50. rectangular region

51. square region
52. axis
53. triangular prism
54. triangular region
55. volume
56. distributive property
57. associative property
58. commutative property
59. factor
60. function
61. opposite-(inverse)
62. reciprocal
63. renaming
64. equivalent statements
65. function rule
66. median
67. prime number
68. mixed numeral
69. rate and ratio
70. greatest common factor
71. arc
72. chord
73. symmetry
74. decimal fraction
75. decimal point
76. place value (less than one)
77. centimeter

MATERIALS - GRADE 5

1. individual number lines
2. place value charts
3. centimeter rulers
4. geometric shapes
5. geometric regions
6. sets of objects to be measured
7. number cards
8. models of simple closed surfaces
9. bead stair
10. square, fractions inset board
11. fraction inlay board
12. number learner set
13. nestling cubes
14. tactile numeral boards
15. geometrical sorting boards
16. building geometric rectangles, triangles, squares
17. cuisenaire rods

IMPORTANT ITEMS

Grade 5

Item	Description	B	M	E
0480 *	Multi-digit multiplication-two factors-regrouping.	M		
0680	Solve problems-two operations.		M	
0650	Dividend less than 10, dividend regrouped, remainder.		M	
0570	Division-inverse of multiplication.		M	
0500 *	More than two factors-regrouping.		M	
0630	Divisor less than 10, no regrouping, remainder.		M	
0510 *	Multiples or powers of ten as factors.		M	
1160	Commutativity-addition of fractions.			M
1200 *	Addition-fractions, unequal denominators.		I	
1190	Common fraction notation-equal denominators.		M	
1300	Subtraction-fractions-unequal denominators.		I	
1370	Commutativity-multiplication-fractions.			I
0640 *	Divisor less than 10, dividend regrouped, no remainder.		M	
1260	Subtraction-inverse of adding fractions.			M
1345 *	Multiplication-fractions-developed from number line.			I
1400	Multiplication-fractions-identity element.			I
1270	Subtraction-fractions-property of zero.			M
3015 *	Expanded notation for fractions.		M	
4070 *	Empty set.	M		
4110	Infinite sets.	I		
3080 *	Read and write to one hundred billion.			M
4000	Meaning of set and member.	I		
4100 *	Finite set.	I		
5040	Recognize two-dimensional figures.			M
5050	Recognize three-dimensional figures.			M
	* Difficult Items			

METHODOLOGY - GRADE 5

Number Systems

0480 Multi-digit multiplication of two factors with regrouping.

Background:

Review expanded notation for renaming numbers.

6316 = 6000 + 300 + 10 + 6

4758 = 4000 + 700 + 50 + 8

Review basic multiplication facts including tables through 12.

Concepts to be developed:

Develop the concept of multiplication using one digit numbers, then proceed to the multiplication by ten or greater. Expanded notation may be used to show regrouping.

$$\begin{array}{rl} 24 = & 20 + 4 \\ & \underline{\quad \times\ 7} \\ & 140 + 28 \quad 168 \end{array} \qquad \begin{array}{rcrcrcl} 34 & = & 34 & + & 34 & & \\ \underline{\times\ 26} & & \underline{\times\ 20} & & \underline{\times\ 6} & & \\ & & 680 & & 204 & = & 884 \end{array}$$

0500 Multiplication with more than two factors with regrouping.

Background:

Knowledge of braces and brackets

Basic multiplication facts

Basic knowledge of the associative property. (3 x 4) = 5 = 3 x (4 x 5)

Concepts to be developed:

Regrouping does not affect the outcome of the multiplication.

Associative property	Distributive property
3 x 8 x 4	2 x 13
(3 x 8) x 4 = 3 x (8 x 4)	2 x (10 + 3) =
24 x 4 = 3 x 32	(2 x 10) + (2 x 3) =
96 = 96	20 + 6 = 26

0510 Powers or multiples of ten as factors.

Background:

Basic knowledge of the tens multiplication facts: $10 \times 10 = 100$ or 10^2, $1000 = 10 \times 10 \times 10 = 10^3$, $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

Concepts to be developed:

The fact that powers are expressed by exponents. For example, 10^1 denotes the tens place, 10^2, the hundreds place, and 10^3, the thousands place.

Thousands	Hundreds	Tens	Ones
$10 \times 10 \times 10$	10×10	10×1	1
10^3	10^2	10^1	10^0

Expansion shows three levels of approach

A. $6 \times 10 = 60$	B. $8 \times 100 = 800$	C. $7 \times 1000 = 7000$
$6 \times 10^1 = 60$	$8 \times 10 \times 10 = 800$	$7 \times 10 \times 10 \times 10 = 7000$
	$8 \times 10^2 = 800$	$7 \times 10^3 = 7000$

0640 Divisor less than ten, dividend regrouped, remainder.

Background:

Knowledge of all multiplication facts.

Experience of division with a one place divisor such as $4\overline{)8}$, $2\overline{)4}$, $5\overline{)10}$

Expression of division as a collection of all facts in addition, subtraction, and multiplication.

Concepts to be developed:

The meaning of regrouping in division is renaming the dividend in various ways.

$4\overline{)49}$ $\quad 4\overline{)40 + 9}$ $\quad 4\overline{)24 + 24 + 1}$ $\quad 4\overline{)20 + 20 + 8 + 1}$

$7\overline{)87}$

$$\begin{array}{r} 10 + 2 \\ 7\overline{)70 + 17} \\ \underline{70 + 14} \\ 3 \end{array}$$

$$\begin{array}{r|l} 6\overline{)149} & \\ 120 & 20 \times 6 \\ \hline 29 & \\ 24 & 4 \times 6 \\ \hline 5 & 24 + R5 \end{array}$$

Place value can also be used.

126	1 hundreds	$100 \div 4 = 25$
	2 tens	$20 \div 4 = 5$
	6 ones	$6 \div 4 = 1 + R2$

1200 Addition with common fraction notation, unequal denominators

Background:

Meaning of the terms numerator and denominator, and the basic structure of fractions.

Review fractional numbers greater than one.

Concepts to be established:

Relate lesson to sets of equivalent fractions.

1/3 = 1/3, 2/6, 3/9, 4/12, 5/15, 6/18

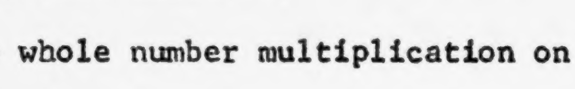
Establish the concept of renaming fractions to have like denominators Refer to addition on number line.

1/3 + 2/9 = 3/9 + 2/9 = 3 + 2/9 = 5/9

7/12 + 3/4 = 7/12 + 9/12 = 7 + 9/12 = 16/12

When the sum is greater than one, relate to mixed numerals with expanded notation.

1345 Multiplication developed from the number line. (fraction)



1/8 + 1/8 + 1/8 = 3 x 1/8 = 3/8

Background:

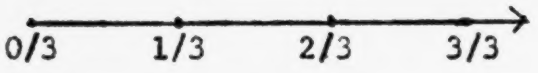
Review the joining of equivalent sets.

Review repeated addition.

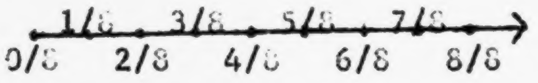
Relate to whole number multiplication on the number line.

Concepts to be developed:

Use the number line to solve the following.



1/3 + 1/3 = 2 x 1/3 = 2/3



5 x 1/8 = 5/8

Use examples of repeated addition involving two like addends and relate to multiplication.

$1/4 + 1/4 = 2/4$; $2 \times 1/4 = 2/4$

$2/5 + 2/5 = 4/5$; $2 \times 2/5 = 4/5$

4100 Finite sets

Background:

Review meaning of sets.

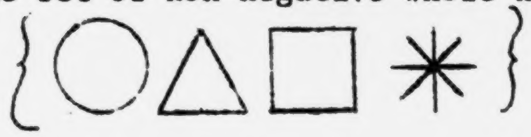
Review the concept of cardinal number of a set.

Knowledge of the symbol called braces $\{\ \}$.

Concepts:

In teaching finite sets begin by matching one to one. Point out that if objects can be paired, the sets are equivalent.

If the elements in a set can be listed and the set has a cardinal number in the set of non-negative whole numbers, it is called a finite set.



$\{$ One Two Three Four $\}$

nA = 4, A is a finite set

B = 3, 4, 5, 6, 7, 8, 9

nB = 7, B is a finite set

C = $\{\ \}$

nC = 0, C is a finite set

4070 Empty set.

Refer to finite sets.

Concepts:

Definition - A set containing no elements is an empty set. Its cardinal number is zero.

Empty braces, $\{\ \}$, are used to indicate the empty set.

Name the elements and the cardinal number of the following:

a. The set of astronauts in this room   Cardinal number = zero

b. The set of students with a $100 bill in his pocket.

Cardinal number = 0.

NUMERATION AND NOTATION

3015 Expanded notation for non-negative rationals.

4 3/6 = 4 + 3/6   5/8 = 4/8 + 1/8

Background:

Basic knowledge of the structure of common fractions.

Knowledge of equivalent common fractions.

Concepts to be developed:

Ability to rename fractions by exploring their structure.

a. 6/8 = 3/8 + 3/8   b. 6/8 = 4/8 + 2/8   c. 6/8 = 5/8 + 1/8

Use the distributive properties to develop expanded notation.

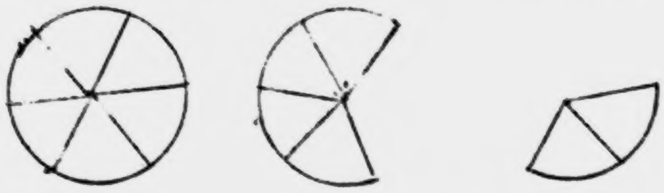
3/2 = 3 x 1/2 = (2 + 1) x 1/2 = (2 x 1/2) + (1 x 1/2) = 1 + 1/2

19/5 = 19 x 1/5 = (15 + 4) x 1/5 = (15 x 1/5) + (4 x 1/5) = 3 + 4/5

Mixed numerals expressed as expanded numerals (a whole number + a fraction).

4 2/3 = 4 + 2/3;   7 1/5 = 7 + 1/5

For slower students use audio-visual devices.



3080 The ability to read and write non-negative whole numbers beyond tens

103 means 1 hundred, 0 tens, and 3 ones.

Background:

Sound knowledge of the Hindu Arabic counting system up to and including millions.

Concepts:

Through a place value chart the extension of values beyond millions and including billions.

Billions			Millions			Thousands			Hundreds		
1	6	7	4	7	3	5	8	1	2	6	4

Read: 7, 402, 136, 401. 7 billion, four hundred two million, one hundred thirty six thousand, four hundred one.

VOCABULARY - GRADE 6

1. associative property
2. closure property
3. commutative property
4. compact numeral
5. cubic centimeter
6. distributive property
7. empty set
8. equal sets
9. superset
10. equation
11. equivalent sets
12. disjoint sets
13. exponent
14. factor
15. finite set
16. infinite set
17. inequality
18. intersection
19. gram
20. kilogram
21. liter
22. union
23. cross product
24. function
25. inverse operations
26. number pair
27. ordered pair
28. arc
29. closed curve
30. closed surface
31. congruent
32. diagonal
33. ellipse
34. endpoint
35. line of symmetry
36. line segment
37. number plane
38. parallelogram
39. perpendicular
40. pyramid
41. quadrilateral
42. rectangular prism
43. rectangular region
44. set of points
45. simple closed curve
46. theorem of Pythagoras
47. triangular prism
48. triangular region
49. vertex
50. volume

51. lattice
52. madian
53. mode
54. rate
55. square root
56. bar graph
57. double graph
58. subset
59. binary numeral
60. common factor
61. common multiple
62. greatest common factor
63. least common multiple
64. prime fractorization
65. prime number
66. expanded numerals
67. joining sets
68. mixed numeral
69. denominator
70. numerator
71. probability
72. separating sets
73. reciprocal
74. set
75. angle bisector
76. loci
77. linear pair
78. perpendicular bisector
79. pi
80. similarity
81. symmetry
82. decimal fraction
83. decimal numeration
84. decimal point
85. scientific notation
86. tenths
87. hundredths
88. thousandths
89. ten thousandths
90. integers
91. not closed
92. rational number
93. replacement set
94. origin
95. central angle
96. circle graphs
97. line graphs
98. pictorial graphs
99. complex fractions
100. edge

101. improper fractions
102. mixed fractions
103. ratio
104. greatest possible error
105. rays
106. composite number
107. cone
108. cylinder
109. decimal
110. decimal equivalent
111. decimal mixed fraction
112. error of measurement
113. lateral area
114. lateral faces
115. metric system
116. acute angle
117. obtuse angle
118. scalene triangle
119. equalateral triangle
120. isosceles triangle
121. polygon
122. longitude
123. parallels of latitude
124. meridian
125. hemisphere
126. simple closed surface
127. bisector
128. prime meridian

MATERIALS - GRADE 6

1. a grid blackboard (green)
2. a ten column abacus
3. wooden geometric kit (models)
4. clock face for clock arithmetic
5. geometric posters
6. compasses
7. protractors
8. place value charts
9. stationary number line
10. visual fractions
11. minute minder
12. individual number line

IMPORTANT ITEMS

Item	Description	B	M	E
1450 *	Division-fractions-binary operation.		I	
1490 *	Division-fractions-inverse of multiplication.			M
1530 *	Division-fractions-non-commutative, non-associative.		I	
1500 *	Division-fractions-closure.		I	
1330	Multiplication-fractions-developed from addition.			M
1370	Commutativity-multiplication-fractions.			M
1390 *	Distributivity-fractions.		I	
1110	Addition-fractions-binary operation.			M
4160 *	Product sets to form new sets.		I	
1200	Addition-fractions-unequal denominators.			M
1210	Decimal fraction notation.			M
4170 *	Partitioning sets to form new sets.	I		
1520	Zero not a divisor-fractions.		I	
1510	Division-fractions-property of one.			M
1430	Common fraction notation.			M
3020	Equivalent common fractions.		M	
3035	Equivalent percent notation.		I	
1090	Ordering-fractions-greater, less, equal.		M	
1230	Subtraction-fractions-developed from subsets.		M	
0670 *	Numbers expressed in exponential form.	I		
8030 *	Graphing-coordinate plane.		I	
1500	Closure-division of fractions.		I	
8040 *	Ordered pairs-coordinate plane.		I	
5230	Construct or draw three-dimensional figures.			I
1310	Decimal fraction notation.			M
	* Difficult Items			

Grade 6

1490 Inverse of multiplying by a fraction.

$$20/6 \times 1/4 = 5/6 \quad \text{or} \quad 5/6 \div 1/4 = 20/6$$

$$9/16 \times 2/3 = 3/8 \quad \text{or} \quad 3/8 \div 2/3 = 9/16$$

Review the process of multiplication of fractions. Refer to divsion as the inverse of multiplication with whole numbers.

Develop the idea that a fraction multiplied by its reciprocal gives a product of 1; thus the term "reciprocal" is introduced.

$$1/2 \times 2 = 1 \qquad 3 \times 1/3 = 1 \qquad 2/5 \times 5/2 = 1$$

Let the student discover that dividing by a number or multiplying by its reciprocal (inverse) produces the same result.

$$5/6 \div 2/3 = 5/6 \times 3/2 = 5/4$$

$$7/8 \div 1/2 = 7/8 \times 2 = 14/8$$

$$15/2 \div 3/4 = 15/2 \times 4/3 = 10$$

1450 Definition of division with fractional numbers: a binary operation.

$$1/2 \div 3/4 = 2/3 \qquad 2/3 \div 5/6 = 4/5$$

Review the meaning of fractional number.

Refer to closure property of multiplication with fractions.

The division of one rational number by another rational number gives a rational number as a quotient.

The operation of division is performed with only two numbers and produces a unique (one and only one) quotient.

$$2/3 \div 7/8 \div 4/5 = 2/3 \times 5/4 = 16/21 \times 5/4 = 20/21$$

1530 Noncommutativity, nonassociativity: division with fractions.

noncommutativity

$$2/9 \div 1/3 \neq 1/3 \div 2/9 \quad \text{since } 2/3 \neq 3/2$$

nonassociativity

$$(2/9 \div 1/3) \div 1/2 \neq 2/9 \div (1/3 \div 1/2)$$

$$\text{since } 2/3 \div 1/2 \neq 2/9 \div 2/3$$

$$1/6 \neq 1/3$$

Review associativity and commutativity with multiplication of fractions.

Work with a problem in division of fractions finding the answer by multiplication. The fact that the answers do not represent equal numbers will prove the noncommutativity and nonassociativity of division.

$2/9 \div 1/3 \neq 1/3 \div 2/9$

$2/9 \times 3 \neq 1/3 \times 9/2$

$2/3 \neq 3/2$

$7/8 \div 4/5 \neq 4/5 \div 7/8$

$7/8 \times 5/4 \neq 4/5 \times 8/7$

$35/32 \neq 32/35$

$6 \div 2/3 \neq 2/3 \div 6$

$6 \times 3/2 \neq 2/3 \times 1/6$

$18/2 \neq 2/18$

$9 \neq 1/9$

$(4/5 \div 2/3) \div 1/2 \neq 4/5 \div (2/3 \div 1/2)$

$4/5 \times 3/2 \times 2 \neq 4/5 \div (2/3 \times 2)$

$6/2 \times 2 \neq 4/5 \times 3/4$

$12/5 \neq 3/5$

$(1/4 \div 5/8) \div 7/12 \neq 1/4 \div (5/8 \div 7/12)$

$(1/4 \times 8/5) \times 12/7 \neq 1/4 \div (5/8 \times 12/7)$

$2/5 \times 12/7 \neq 1/4 \times 14/15$

$24/35 \neq 7/30$

1500 Closure, a property of division.

Remember that the quotient of 2 whole numbers is the whole number which when multiplies by the divisor gives the quotient.

quotient x divisor = dividend

$6 \div 2 = 3$ so $3 \times 2 = 6$

A teacher may express the above example in a general way such as $a \div b = c$ implying that $c \times b = a$.

The student should know that a fractional number is the result of comparing the cardinal number of two sets to the more abstract definition of a rational number.

Division by zero is undefined, that is, we cannot name a quotient when the divisor is zero.

The student should understand that when a set of 6 objects is separated into 2 equivalent subsets, we are finding one-half of the set and the cardinal number of each subset may be named in these four ways: $6 \div 2$, $6 \times 1/2$, $2\overline{)6}$, $6/2$. The question, "When a rational number is divided by a rational number, will the result always be a rational number?", should be proposed to the student. The student should recognize intuitively that the answer to the question is yes. Even when the example is one such as $7/4 - 3/4 = \boxed{}$, the solution may be expressed as the rational number 4/4.

1390 Distributivity of multiplication over addition. (fractions)

$$2/3 \times (\tfrac{1}{2} + \tfrac{1}{4}) = (2/3 \times 1/2) + (2/3 \times 1/4)$$

$$2/3 \times 3/4 = 1/3 + 1/6$$

$$1/2 = 1/2$$

Understanding property of distributivity with whole numbers. Have concept of meaning and use of parentheses for grouping. Understand the use of common denominators for the addition of fractions.

Distributive rule for multiplication: The sum of 2 addends with a common factor may be named by multiplying the sum by the common factor or by naming the two products and then adding.

$$5 \times (3 \not- 2) = (5 \times 3) \not- (5 \times 2)$$

$$5 \times 5 = 15 \not- 10$$

$$25 = 25$$

$$2/7 \times (3/4 \not- 5/8) = (2/7 \times 3/4) \not- (2/7 \times 5/8)$$

$$2/7 \times (11/8) = 3/14 \not- 5/28$$

$$22/56 = 11/28$$

4160 The ability to form new sets by product sets.

$A = \{x, y\}$ $B = \{1, 2, 3\}$

The product set is $\{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)\}$

The cardinal number of the Cartesian product is 6, (2 x 3)

Review the understanding of sets, cardinal number, and ordered pairs, and one-to-many correspondence.

Set the goals of the development of the concept of an ordered pair and the cross product of two sets. Introduce the idea that multiplication may be used to find the cardinal number of the cross products of two sets. Finding the number of elements in the cross products of two sets is one of the applications of multiplication. The other application is that of repeated addition. Patterns should be presented that relate the two applications. Speak of a pair of things usually considered as a single object. For example on a baseball team the pair of players made up of a pitcher and a catcher is a battery.

We often agree to name the objects in a pair in a specified order. Thus if we agree to list the players in a battery in the order (pitcher, catcher), then the ordered pair (John, Bill) specifies that John is a pitcher and Bill is a catcher. Explain the need for agreeing on an order of listing the names. Students may begin by choosing pairs in random fashion. Suggest that if the batteries are listed in an organized way there will be less chance of missing a pair. Introduce the notation $P \otimes C$ for the set of ordered pairs.

$$P = \{\text{John, Mike, Harry}\} \qquad C = \{\text{Pete, Joe}\}$$

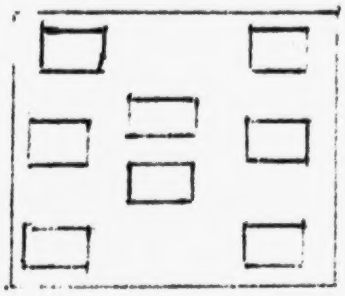
$$P \otimes C = \{(\text{John, Pete}), (\text{John, Joe}), (\text{Mike, Pete}), (\text{Mike, Joe}), (\text{Harry, Pete}), (\text{Harry, Joe})\}$$

$$n(P) = 3,\ n(C) = 2,\ n(P) \times n(C) = 3 \times 2 = 6$$

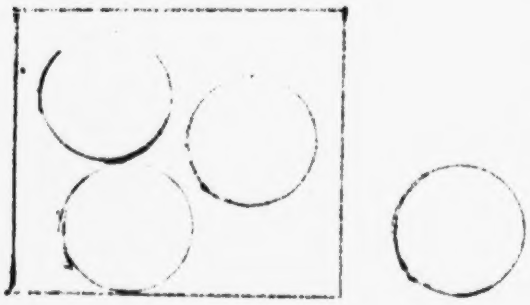
$$\{a, b\} \otimes \{4, 5, 6\} = \{(a, 4), (a, 5), (a, 6), (b, 4), (b, 5), (b, 6)\}$$

4170 The ability to form new sets by partitioning sets.

Separating sets into subsets.



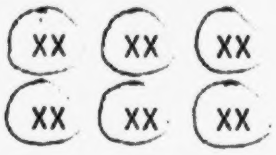
You have a set of 8 boxes. Separate the set into subsets of 2 each. How many such subsets do you find?

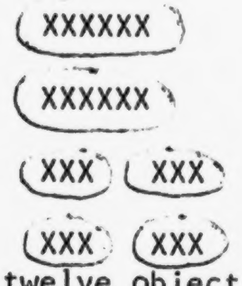


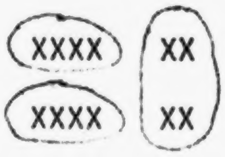
You have a set of four circles. One circle is removed, what set remains or how many circles remain?

Review set and subset.

Concepts: Division of sets into selected subsets.







A set of twelve objects can be partitioned into subsets each containing 2 elements, 6 elements, 3 elements, or 4 elements.

0670 Numbers expressed in exponential form. Division of whole numbers.

$$7^3 \div 7^2 = \frac{7 \times 7 \times 7}{7 \times 7} = 7^{3 - 2}$$

$$2^4 \div 2 = \frac{2 \times 2 \times 2 \times 2}{2} = 2^{4 - 1} \quad \text{or} \quad 2^3$$

Review the use of parentheses and the meaning of exponents. The student should have an understanding of the multiples or powers of ten.

After the students fully understand the meaning of exponents, the introduction of the way in which exponents may be used to simplify division may be introduced.

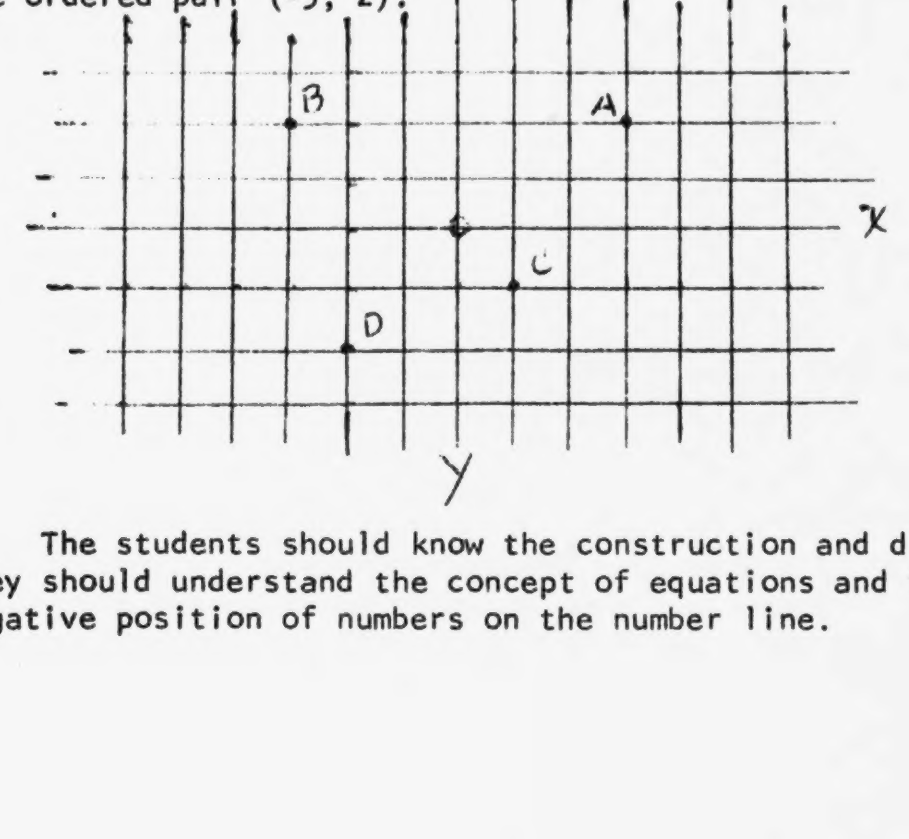
$$630 \div 90 = \left[(6 \times 100) + (3 \times 10)\right] \div (9 \times 10)$$

$$\left[(6 \times 10^2) + (3 \times 10)\right] \div (9 \times 10)$$

$$\frac{6 \times 10 \times 10}{9 \times 10} + \frac{3 \times 10}{9 \times 10} = \frac{6 \times 10 + 3}{9}$$

8030 Ordered pairs on a coordinate plane.

The ordered pair (x, y) will locate points on the plane where the x direction is horizontal and the y direction is vertical. The point A is located by the ordered pair (3, 2). The point B is located by the ordered pair (-3, 2).



The students should know the construction and division of a grid. They should understand the concept of equations and the positive and negative position of numbers on the number line.

Present a diagram of a grid on the board explaining the terms x axis, y axis and origin. The origin is the point where the horizontal and vertical lines intersect. Explain that the location of any point on the plane is made by a reference to both of these number lines. Have some student assist in drawing a number plane on the chalkboard and provide each student with a sheet of graph paper. Give the students a teacher prepared list of ordered pairs to locate on the plane. Examples such as: (3, 6), (5, 2), (7, 4), (5, 8), (0, 0), (-3, -1).

Finding functions through equations and putting them on the coordinate plane. A function is a set of ordered number pairs and a rule that relates the numbers of each pair such that for each first member of a pair there is one and only one second member. If x names the first member and y names the second member of a pair, then frequently an equation describing the rule for x and y may be used to represent a function (relate to solutions on a coordinate plane). One idea they may discover is that if two graphs intersect, the ordered pair for the point of intersection gives the only pair of numbers that is a solution of both equations involved.

Use the equations to complete the tables.

$x - 3 = y$ $4 - 3 = 1$, therefore, if $x = 4$, $y = 1$

x	4	5	6	10	3	0
y	1	a	b	c	d	e

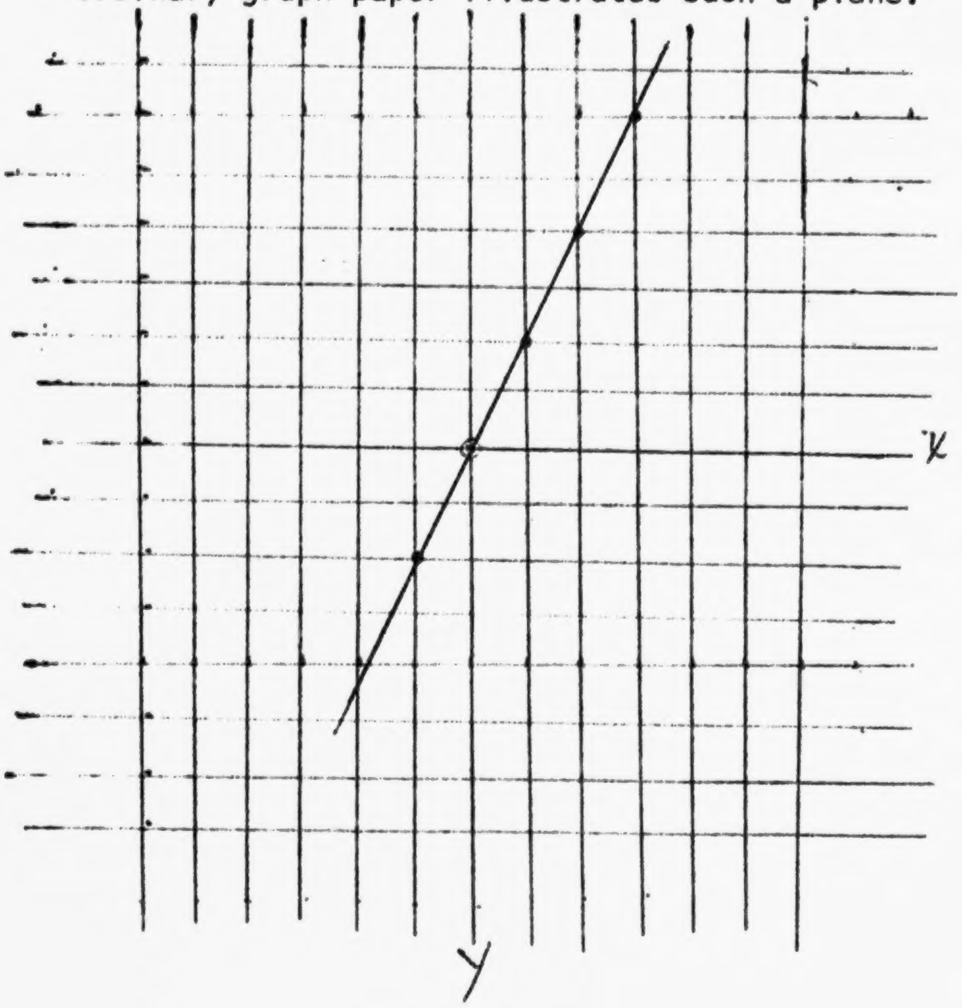
$(3x) + 2 = y$

$(3 \times 3) + 2 = 11$, if $x = 3$; $y = 11$

x	y
2	8
3	11
4	a
0	b
1	c
-1	d

8040 Solution sets of equalities and inequalities on a coordinate plane.

A plane (surface) in which pairs of numbers are used to locate points is referred to as the Cartesian or Coordinate plane. Ordinary graph paper illustrates such a plane.



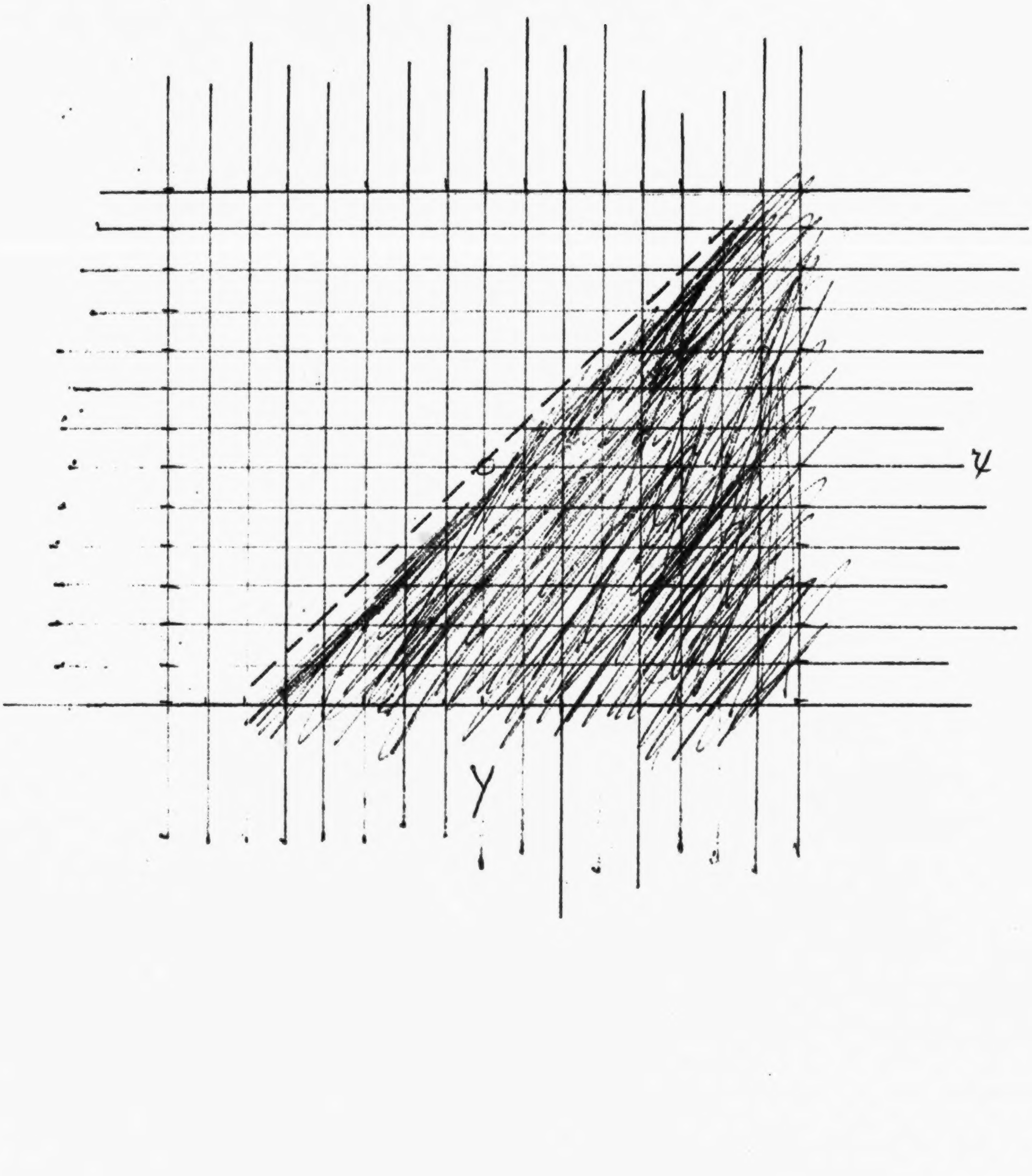
Equality

$y = 2x$

x	y
-1	-2
0	0
1	2
2	4
3	6

The solution of a number sentence is a number whose name makes the sentence true. Should know and understand concepts of equality, inequality, and equations. Know construction and divisions of a grid. Basic ideas of a number line, positive and negative position of numbers. The idea of functions.

The graph of the equation $x = y$ is the line on the grid. What would be the graph of $x > y$? Consider the pairs (4, 2) and (6, 3). They do not satisfy the equation $x = y$ but they do satisfy the inequality $x > y$ since $4 > 2$ and $6 > 3$ are true statements. In fact any point in the shaded region of the shown numbered plane will correspond to an ordered pair which will satisfy $x > y$. So the graph of the inequality $x > y$ is all of the points in the shaded region of the numbered plane.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TITLE III, ESEA
THE TEACHING OF MODERN MATHEMATICS, K-6
CALIFORNIA AREA SCHOOL DISTRICT

1969-1970 Instructional Model

The instructional model was incorporated into the pilot school to enhance the mathematics curriculum guide developed by the elementary teachers of the California Area School under ESEA Title III. This model for instruction includes the use of pre-tests, post-tests, pre-requisites and behaviors as they relate to the innovative idea of flexible grouping in mathematics.

The pre-tests, developed by the teachers, are based on the expected content behaviors. The results of these tests are used to group the students according to their individual needs for instruction in specific behaviors. These same tests are used again as post-tests to evaluate the progress of the students whose pre-test scores indicated a need for basic instruction.

The pre-requisites for each content items are reviewed as a class activity before the pre-test is administered. After the tests are scored, the students are divided into three groups: the basic group, the selected group, and the extended group. The basic group includes those students who need instruction in all or almost all of the expected behaviors. The selected group is that group of students who require instruction in only a few specific behaviors. It is within this group that the instruction is, of necessity, almost

SE 010 448

individualized instruction. The extended or optional group includes those students whose pre-test scores show a mastery of ninety percent of the expected behaviors for that particular content item. This group is mainly self-directed and is involved with optional content items listed in the curriculum guide.

The group to which each student is assigned may vary with each new concept. To insure maximum instruction for each group, a flow-chart has been developed. This chart indicates the flexible grouping for each student as mastery of the content behaviors increases. A management cycle is also important for meaningful instruction. This form shows a recommended time schedule and the use of the instructor's guidance within each group.

Included as part of the instructional model is an open ended teachers manual. This is a master file of materials for each of the twenty-five content items specified in the curriculum guide. Included in the manual are instructional needs such as lesson plans, supplementary worksheets, pre-requisites and essential content behaviors, pupil response sheets, audio-tapes, enrichment or optional materials, and individual record sheets. Most of these instructional materials have been formulated by the individual and therefore reflects her methods and techniques.

The students are not confused by a multitude of changes in instruction and they are more at ease when they hear a familiar voice on the audio-tapes.

The individual record sheets are used to help evaluate the progress of the students. They show a complete picture of each students content achievement by listing both pre-test and post-test scores for each expected behavior under each content item. They are also a help in evaluating effectiveness of the instructional model by recording the percent of students who increase their scores at the end of the instructional cycle.

Dissemination of the overall program has taken various paths throughout the year. The curriculum guide and the instructional model have been discussed at teacher in-service sessions both locally and within the tri-county region.

Personnel involved in this type of dissemination included teachers from the pilot school, the Elementary Supervisor, the coordinator of the project, the outside consultant, and the director from the regional office.

Copies of the curriculum guide and outlines of the instructional model have been distributed to numerous school districts and colleges to be included in their curriculum libraries. In addition, visitors to the pilot school have observed the use of the model in the classroom.

Student teachers, who will eventually carry the basic ideas of the program into their own classrooms, have been involved in developing audio-tapes and enrichment materials for use within the structure of the program. They have helped with the distribution of materials when the regular teacher was devoting her time to individual groups under the management cycle. In addition, the student teachers participated in the general in-service session and in a "close-circuit" session chaired by the consultant for the program.

For the summer of 1970, it is hopeful that revisions and/or improvements of the instructional model can be made. The list of twenty-five (25) important content items, the analysis for reliability and relevence of pre-tests, enrichment and optional materials, and development of audio-tapes for basic instruction are some of the more important areas of consideration.

The project is to be extended fully to other district schools. Development of materials for Teacher Manuals, purchasing of necessary equipment and materials, in-service for all teachers involved, worksheets, tapes, manipulative materials for specific behaviors, and practice materials for slower students are areas to be included in future developments.

SE 010 448

UNIT INSTRUCTIONAL FLOW CHART FOR FLEXIBLE GROUPING IN ELEMENTARY MATHEMATICS

CALIFORNIA AREA SCHOOL DISTRICT

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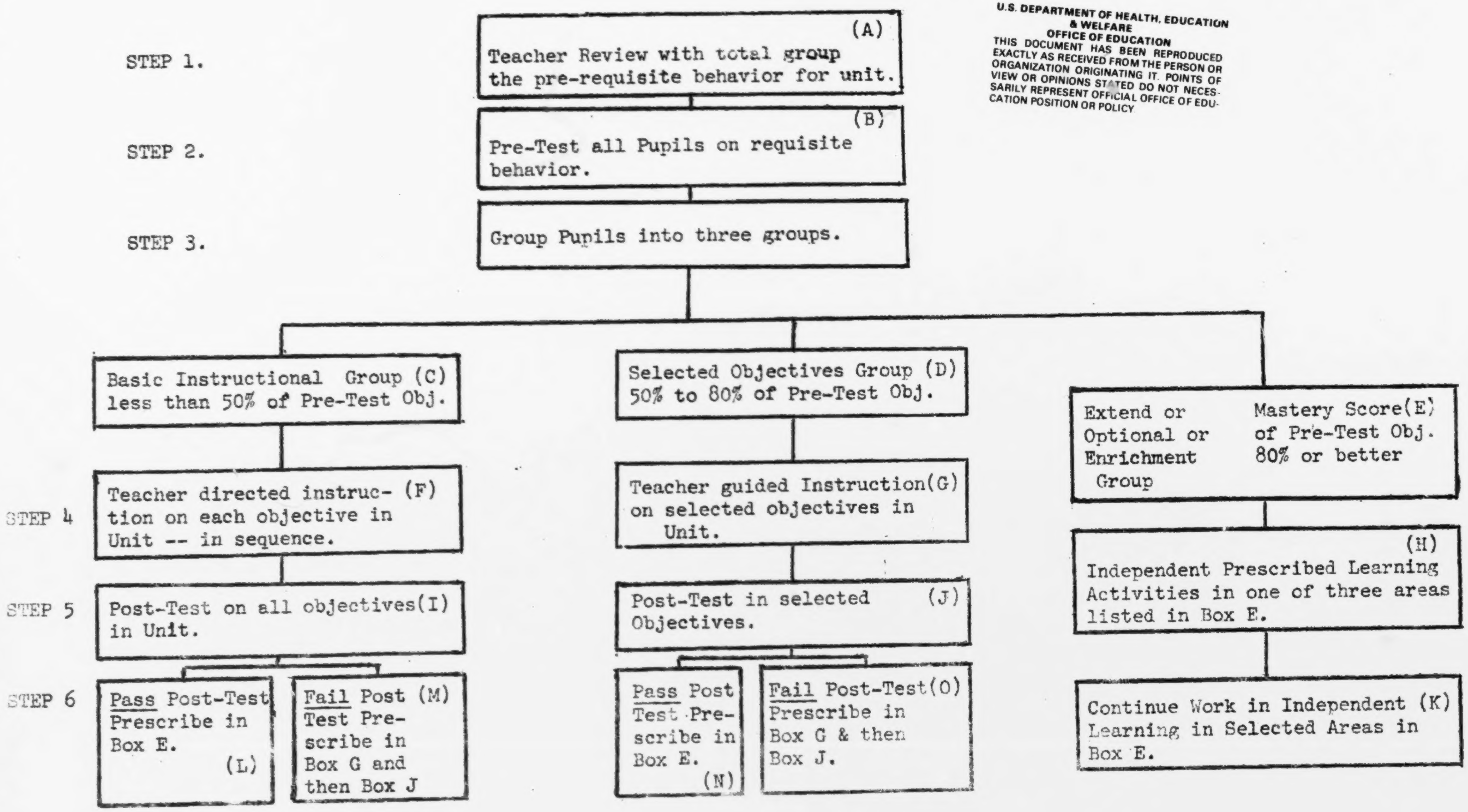

STEP 1. (A) Teacher Review with total group the pre-requisite behavior for unit.

STEP 2. (B) Pre-Test all Pupils on requisite behavior.

STEP 3. Group Pupils into three groups.

STEP 4

Basic Instructional Group (C) less than 50% of Pre-Test Obj.

Teacher directed instruction on each objective in Unit -- in sequence. (F)

Selected Objectives Group (D) 50% to 80% of Pre-Test Obj.

Teacher guided Instruction (G) on selected objectives in Unit.

Extend or Optional or Enrichment Group

Mastery Score (E) of Pre-Test Obj. 80% or better

(H) Independent Prescribed Learning Activities in one of three areas listed in Box E.

STEP 5

Post-Test on all objectives (I) in Unit.

Post-Test in selected Objectives. (J)

STEP 6

Pass Post-Test Prescribe in Box E. (L)

Fail Post Test Prescribe in Box G and then Box J (M)

Pass Post Test Prescribe in Box E. (N)

Fail Post-Test (O) Prescribe in Box G & then Box J.

Continue Work in Independent (K) Learning in Selected Areas in Box E.

NOTES

A. The time interval for each unit is contingent upon:

  a. Number of objectives in the unit. However, one week of instruction is the anticipated average time interval for a unit of instruction.

  b. Time required for Basic Instructional Group to achieve mastery of the objectives within the Unit.

  c. Overall classroom management problems.

B. Criterion for determining Group Placement based on Pre-Test Score:

  1. If a pupil passes less than 50% of the objectives (to pass an objective, a pupil must score at least 80%), he is placed in the basic group.

  *2. If a pupil passes between 50% and 80% of the objectives, he is placed in the Selected group.

  *3. If a pupil passes 80% or more of the objectives on a Pre-Test, he is placed in the Ex-Op-Er group.

  Less than 100% of the objective -- Place pupil in Extended Activity.

RECOMMENDED UNIT TIME -- MANAGEMENT CYCLE FOR FLEXIBLE GROUPING IN ELEMENTARY MATHEMATICS

CALIFORNIA AREA SCHOOL DISTRICT

DAY	TIME	MANAGEMENT SYSTEM		
1st Day	40 "	Review Pre-requisite Skills with Total Group		
2nd Day	40 "	Administer Pre-Test for Unit to all pupils. (Score & Assign to Groups before 3rd day)		
		GROUPS		
		Basic	Selected	Ex-Op-Er
3rd Day	40"	Teacher Directed Study	Independent Study	Independent Study
		Independent Study	Teacher Guidance	Independent Study
		GROUPS		
		Basic	Selected	Ex-Op-Er
4th Day	40"	Independent Study	Independent Study	Teacher Guidance
		Teacher Directed Study	Independent Study	Independent Study
		GROUPS		
		Basic	Selected	Ex-Op-Er
5th Day	40"	Independent Study	Teacher Guidance	Independent Study
		Post - Test	Post - Test	Independent Study

NOTES

(1) As a pupil in the selected group masters all selected objectives (as measured by a post-test on these objectives), he is assigned to the Ex-Op-Er group within the unit time cycle.

(2) As an individual pupil's rate of progress in the basic group increases, he may also be assigned to the Ex-Op-Er group.

(3) The classroom teacher may need to extend the middle section of this cycle, based upon rate of pupil progress in the basic instructional group and the scores obtained on post-tests.

(4) The teacher may decide to teach the total group an optional unit. The optional unit would be scheduled within the content sequence guide developed by the school system.

(5) The 40 minute interval may need to be scheduled in two twenty minute sessions for Kindergarten and 1st grade pupils.

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CALIFORNIA
AREA SCHOOL DISTRICT
California, Pennsylvania 15419

SE 010 448

Primes Math

OV 12 1970

The curriculum guide is a vital part of the program since it has the complete course of study as planned K-6 for the new mathematics program. This will make it possible to have uniformity throughout the district.

Multi text approach

1. The books being used as basic texts are as follows:
    - (a) Modern Arithmetic Through Discovery, Silver Burdett Co., 1965
    - (b) Elementary Mathematics Patterns and Structure, Holt, Rinehart, and Winston, 1968
    - (c) Modern School Mathematics Structure and Use, Houghton Mifflin Co., 1967

Multi Flexible grouping

1. Pre-requisites are taught for each content item.
2. Pre tests are given which were made by each classroom teacher for his or her respective grade. There is a pretest for each behavior within the content item.
3. Grouping is determined by the following scale:
    80 - 100% Extended, Optional, or Enrichment group
    50 - 80% Selected group
    0 - 50% Basic group

4. Teaching of these groups
    - (a) 0 - 50% receive basic instruction on all behaviors of the content item.
    - (b) 50 - 80% are taught those behaviors which were missed.
    - (c) 80 - 100% are given enrichment, or extended or optional materials.

5. Post test given those in the basic and selected groups.

6. Evaluation progress records are kept for each child on each content item.

A specified order of content items and when they should be taught during the school year has been a part of the program.

Equipment

1. An 8 station listening center has been placed in each classroom K-6 enabling the teacher to use a tape recorder which was provided to facilitate teaching several groups simultaneously.

2. Tapes have been made by each teacher for each content item. They are classified as:
    - (a) teaching tapes
    - (b) drill tapes
    - (c) extended activities
    - (d) optional activities
    - (e) enrichment activities

3. S.R.A. drill tapes are also available for use for many content items.

Individualized instruction to some degree.

Kindergarten
Primes Math

I. Basic Texts

A. Modern Mathematics Through Discovery, Silver Burdett Company, 1968.

B. Greater Cleveland Mathematics Program, Science Research Associates, Inc., 1962.

C. Modern School Mathematics, Structure and Use, Houghton Mifflin Company, 1967.

II. Resource Texts

A. Let's Begin, Harcourt, Brace & World, Inc., 1966.

B. Elementary School Mathematics, Addison-Wesley Publishing Company, 1968.

C. Developing Pre-Number Ideas, Holt, Rinehart and Winston, Inc., 1965.

D. Sets and Numbers (K) L.W. Singer Company, 1968.

E. Beginning New Arithmetic, McCormick-Mathers Publishing Company, 1968.

F. Elementary School Mathematics, Addison-Wesley Publishing Company, Inc., 1963.

G. Mathematics-Sets-Numbers-Numerals, Laidlaw Brothers Publishing, 1968.

H. Modern Arithmetic Through Discovery, Silver Burdett Company, 1964.

Kindergarten
Primes Math

I. Items to be mastered

  A. None

II. Items to be introduced

  A. Representations of point, line, plane, and space

    1. Pre-requisites

      (a) Inside and outside of a simple closed figure

      (b) Counts number of sides, right angles or angles.

    2. Behaviors to be taught

      (a) Curved line

      (b) Straight line

      (c) Polygon

      (d) Plane

      (e) Space

      (f) Point

    3. Enrichment

      (a) None

  B. Curves, simple, closed, and open

    1. Pre-requisites

      (a) Matching of shapes appropriately according to form and color

      (b) Completes pattern matching activities which involve primitive concepts of set identification, union and intersection.

    2. Behaviors to be taught

      (a) Geometric shapes

      (b) Curves

      (c) Line segments

      (d) Responds to directions using the words inside, outside, on, on the boundary

3. Enrichment

(a) None

C. Two-dimensional figures (plane)

1. Pre-requisites

(a) Reviews matching shapes according to form only

2. Behaviors to be taught

(a) Identifies the figure and gives a name for a circle, triangle, square, and rectangle.

3. Enrichment

(a) Game on tape — "Spotting the Shape"

D. Geometric figures in environment

1. Pre-requisites

(a) Locating in pictures the relationships high, low; hear, far; nearest, farthest; big, little etc.

2. Behaviors to be taught

(a) Matching of shapes which are the same

3. Enrichment

(a) None

E. Spatial relations without.measurement (size, position)

1. Pre-requisites

(a) Identifying in pictures the relationship smallest, largest, more, fewer, most, and fewest

2. Behaviors to be taught

(a) Locating in pictures the relationships high, low; near, far; nearest, farthest; big, little etc.

3. Enrichment

(a) None

F. Equal Sets

1. Pre-requisites

   (a) Identifies the correct object in a picture when asked to "find the picture of a set of ______ "When a common property is named".

2. Behaviors to be taught

   (a) Sets which are equal or identical.

3. Enrichment

   (a) Use game on tape "Spot the Set"

G. Geometric designs or patterns (sequences)

1. Pre-requisites

   (a) Reproduces simple spatial patterns composed of two elements

2. Behaviors to be taught

   (a) Continuing simple spatial patterns composed of two elements.

   (b) Reporduces and solves spatial patterns using up to four elements

3. Enrichment

   (a) None

H. Non-equivalent sets

1. Pre-requisites

   (a) Matching objects in pictured sets by drawing lines and responds that one set has more, less, or same number as a given set. Sets to five.

   (b) Pairs picture of two sets by drawing connecting lines. Sets to 10.

   (c) Adds to a second set or draws a second set of objects. Sets to 10.

2. Behaviors to be taught

   (a) Compares sets by visual inspection. Sets of 10.

   (b) Identifies in pictures the relationship more, fewer, most, fewest etc.

3. Enrichment

    (a) None

I. Developing cardinal numbers one thru ten

1. Pre-requisites

    (a) Reviews a pictured set for numbers 1-5

    (b) Draws an appropriate set when presented with numeral one to five.

2. Behaviors to be taught

    (a) Introduces abstracting the cardinal number from pictures of structured groups up to five in number. Responds by giving a no. word name.

    (b) Pairs of objects in two given sets. Sets to 10.

    (c) Identifies a set which matches; draws or completes a set to match a given numeral 0-10

3. Enrichment

    (a) Story on tape "Chicken Little Counts to Ten" by Margaret Friskey

J. Ordering numbers and objects as greater than; less than; equal to

1. Pre-requisites

    (a) Identifies the cardinal no. of a structured group to 10 and responds by selecting a numeral.

    (b) Matches objects in pictured sets by drawing connecting lines and responds that one set has less, more, or same no. as a given set. Sets of five.

2. Behaviors to be taught

    (a) Pairs off objects in two given sets. Sets to 10 in number.

    (b) Identifies in pictures the relationship more, fewer, most, fewest

    (c) Identifies what no. comes just after or just before a given no. or between two given numbers. for numbers to 10.

3. Enrichment

    (a) Tape, Writing numerals for inequalities.

K. Equivalent (sets) one-to-one correspondence

1. Pre-requisites

(a) Identifies the correct object in a picture when asked to "find the picture of a set of __ __ __" a common property is named.

2. Behaviors to be taught

(a) Pairs pictures of two sets by drawing connecting lines. Sets to 10.

(b) Identifies a set as having more, fewer or, the same number of objects. Sets to 10.

L. Associating idea of number with number line (1-1 correspondence)

1. Pre-requisites

(a) Identifies the cardinal number of a structured group to 10. Responds by selecting a numeral.

(b) Identifies a pictures or set containing none or not any when presented with the symbol for zero.

2. Behaviors to be taught

(a) Identifies what number comes just after or just before a given number or between two given numbers, for numbers to 20. Responds by selecting the correct number.

3. Enrichment

(a) None

M. Counting to find cardinal number of set (1-1 correspondence)

1. Pre-requisites

(a) Identifies what number comes just after or just before a given number or between two given numbers, for numbers to 10. Responds by selecting the correct number.

2. Behaviors to be taught

(a) Counts sets of from one to five objects. Identifies the largest number as cardinal no. of the set. Matches each object with correct progression of printed numerals.

(b) Identifies a set which matches a written number word for numbers to 10.

3. Enrichment

(a) None

N. Role of one in addition

1. Pre-requisites

(a) Identifies the no. in each of a number of sub sets and the number there are altogether for sums of structured groups up to 10.

(b) Identifies what number comes just after or just before a given number or between two given numbers, for numbers to 10.

3. Enrichment

(a) Story on tape — "Twenty-Two Bears" by Claire Huchet Bishop

O. Reading and/or writing words or numerals for the ten basic symbols (0-9)

1. Pre-requisites

(a) Abstracts the cardinal number from pictures of structured groups up to 10 in number. Responds by selecting a numeral from a choice of numerals.

(b) Identifies a picture of a set containing none or not any when presented with the symbol for zero.

2. Behaviors to be taught

(a) Supplies a pictured set for numbers 1-5 when presented with numerals 1-5 draws an appropriate set.

(b) Identifies a set which matches; draws or completes a set to match a given numeral, 0-10.

3. Enrichment

(a) None

P. Backward-rote counting

1. Pre-requisites

(a) Identifies what number comes just after or just before a given number or between two given numbers to 10.

2. Behaviors to be taught

(a) Identifies natural order of the whole numbers using the one less pattern.

3. Enrichment

(a) Story on tape —" Seven Missing Chicks" by Catherine Barr

Q. Meaning-set and member (elements)

1. Pre-requisites

(a) Completes pattern matching activities which involve primitive concepts of set identification, union, and intersection. Matches shapes according to color and form.

2. Behaviors to be taught

(a) Identifies the correct object when asked to "find the picture of a set of _____when a common property is named.

3. Enrichment

(a) None

R. Developing ordinal number sense

1. Pre-requisites

(a) Identifies in pictures the relationship before, after

(b) Identifies in pictures the relationship before, after

2. Behaviors to be taught

(a) Locates the requested ordinal position, first to tenth, in an ordered set of objects.

3. Enrichment

(a) Story on tape —"Two lonely Ducks" by Roger Duvoisin

S. Ordinal counting and/or sequence of numbers

1. Pre-requisites

(a) Locates the requested ordinal position, first to tenth, in an ordered set of objects.

2. Behaviors to be taught

(a) Given dot pictures with on sequential numeral per dot, connects dots in correct order. Numbers to 10.

(b) Identifies what number comes between two given numbers and what number comes just after or just before a given number.

3. Enrichment

(a) None

T. Developing cardinal number zero

1. Pre-requisites

(a) Identifies a set which matches; draws or completes a set to match a given number, 0 to 10.

2. Behaviors to be taught

(a) Identifies a picture or set containing none or not any when presented with the symbol for zero.

3. Enrichment

(a) None

U. Subsets

1. Pre-requisites

(a) Identifies the correct object in a picture when asked to "find the picture of a set of _ _ _" when a common property is named.

2. Behaviors to be taught

(a) Identifies a subset of a given set

3. Enrichment

(a) None

V. Commutativity, a property of addition

1. Pre-requisites

(a) Identifies the number in each of a number of subsets and the number there are altogether for sums structured groups up to 10.

2. Behaviors to be taught

(a) Identifies the order in which the sets were joined to make a new set

3. Enrichment

(a) Story of tape —— "Mystery of Three Fires" by Margaret Frisleey

W. Addition developed from union of disjoint sets or joining action

1. Pre-requisites

(a) Identifies the cardinal number of a structured group to 10. Responds by writing or selecting a numeral

2. Behaviors to be taught

   (a) Identifies the number there are in each a number of subsets and the number there are altogether, when given up to five objects.

   (b) Identifies the number in each of a number of subsets and the number there are altogether for sums of subsets and the no. there are for sums of structured groups up to 10.

3. Enrichment

   (a) None

III. Optional Items

A. None

First Grade
Primes Math

- I. Items to be mastered in Grade I
  - A. Counting the cardinal number of a set.
    - 1. Prerequisite behaviors
      - a. Ability to identify an empty set written with set braces.
      - b. Ability to count sets from one to five objects by drawing lines to succeeding numerals on a number line; then identifying the largest number as the cardinal number of a set.
    - 2. Behaviors to be taught
      - a. Abstracts the cardinal number from pictures of structured groups to five, and gives the number word name.
      - b. Gives the cardinal number of a structured group to 12.
      - c. Identifies a matching set and can craw or complete a set to match numerals to 12.
      - d. Ability to count objects to lw, writing a numeral for each object.
      - e. Identifies a set to match written number words for numbers to 20.
      - f. Identifies tens and ones in pictures of numbers to 200 by writing or selecting the correct numeral.
      - g. Identifies cardinal number of structured group to 200 by writing numeral.
    - 3. Enrichment activities
      - a. Use count-a-ladder activity.
      - b. Use pegboard and pegs to illustrate structured group. Have teaching tape available for this type activity.
  - B. Developing cardinal number 1-10.
    - 1. Prerequisite behaviors
      - a. Can pair off objects in two sets up to 12.
      - b. Can pair off pictures of two sets by drawing connecting lines-sets to 12.
    - 2. Behaviors to be taught
      - a. Abstracts cardinal number from pictures of groups to five; gives oral name or selects matching group or selects correct numeral from a choice of numerals.
      - b. Can identify a matching set, and can draw or complete a set to match a given set or a given numeral from 0 to 12.
      - c. Identifies the cardinal number of a structured group to 12 by responding orally, or by writing or selecting a numeral or a matching set.
    - 3. Enrichment activities
      - a. Flannel-board activities for building sets and matching them.
  - C. Reading and/or writing words or numerals for the ten basic symbols (0-9).
    - 1. Prerequisite behaviors
      - a. Can match objects in sets by drawing connecting lines, and can see that one set has more, has less, or has the same number as a given set—sets to 5.
      - b. Pairs off objects in sets to 12.
      - c. Pairs pictures of sets by connecting lines—sets to 12.
    - 2. Behaviors to be taught
      - a. Can draw an appropriate set for numerals 1 to 5.
      - b. Writes numerals 0 to 10 in forward or reverse order.
      - c. Identifies matching sets, and can draw or complete a set to match numerals 0 to 12.

        d. Identifies a set containing none when presented with zero symbol.
        e. Identifies a group which matches a given number word for 1 to 5.
        f. Identifies a set which matches a written number word to 10.
        g. Responds to printed directions containing number words to 10.

II. Items to be introduced in Grade I.
  A. Understanding of the empty set.
    1. Prerequisite behaviors
        a. Can write numerals 0 to 10 in forward or reverse order.
        b. Can identify a matching group for number words 1 to 5.
        c. Can abstract a cardinal number and write numeral for groups up to 5.
    2. Behaviors to be taught
        a. Can identify a set with none or not any when shown the symbol for zero.
        b. Can identify an empty set written with set braces.
        c. Can orally give cardinal number of set to 12.
        d. Can write or select cardinal number of set to 12.

  B. Developing cardinal numbers beyond ten.
    1. Prerequisite behaviors
        a. Identifies picture or set of none when shown a zero symbol.
        b. Writes or selects a numeral for a group to 12.
        c. Identifies matching sets, and can draw or complete a set for numerals 0 to 12.
    2. Behaviors to be taught
        a. Identifies place value of units and tens in numbers 10 to 20 by drawing pictures.
        b. Writes numerals to 99, as ___tens,___units.
        c. Uses expanded notation to 99.
        d. Can write missing numerals in sequence from 1 to 200.
        e. Can write or select numeral to 200 from pictures showing tens and ones.
        f. Identifies number of tens in any multiple of ten to 200.

  C. Developing understanding of whole numbers; units, tens (10-99)
    1. Prerequisite behaviors
        a. Can write numerals 0 to 10 forward and reverse.
        b. Knows matching sets; can draw or complete a set 0 to 12.
        c. Identifies number of tens in numerals to 200.
    2. Behaviors to be taught
        a. Identifies place value in numerals 10 to 20.
        b. Writes numerals from 0 to 20.
        c. Writes numerals to 99 as___tens___units.
        d. Writes missing numerals in sequences from 1 to 200.
        e. Knows and writes cardinal number of groups to 200.
        f. Writes or selects numeral represented by pictures to 200.

  D. Elementary facts of addition
    1. Prerequisite behaviors
        a. Identifies number in each subset and number altogether for sums to 12.
    2. Behaviors to be taught
        a. Can construct or complete sets for sums to 12, when given a number sentence with + and = signs.
        b. Can provide or complete an addition sentence using + and = for pictures of sets or objects ___sums to 12.

c. Finds sum of two or more addends by using number line-sums to 20.
d. Finds sum of two or more addends by writing numerals-numbers to 20.
e. Solves addition problems written in horizontal form without pictures to 12.
f. Solves addition problems written vertically without pictures-to 12.
g. Can match addition expressions with numerals with pictured groups available.
h. Can solve one-step word problems with pictures, using addition facts to 12.
i. Constructs or completes an addition table (12x12).
3. Enrichment activities
a. Cross number puzzles
b. Cross number puzzles on tape.
c. Count-a-ladder

E. Addition developed from number line.
1. Prerequisite behaviors
a. Can write and count numerals from 0 to 20.
2. Behaviors to be taught
a. Can use commutative property of addition by completing two number sentences for a picture.
b. Can find sum of 2 or more addends by using number line (sums to 20), both orally and by writing numerals.
c. Can add horizontal and vertical problems without regrouping. (sums to 200)
3. Enrichment activities
a. SRA drill tapes
b. Number line activities

F. Subtraction developed in relation to subsets or separating action.
1. Prerequisite behavior
a. Can ring subsets of a given set.
2. Behaviors to be taught
a. Can"take away"by removing objects and tell how many are left.
b. Can complete a subtraction equation using — and = signs from pictured sets.
c. Can distinguish and write either an addition or subtraction equation from given pictured situations.
d. Finds remainders for subtraction exercises.
e. Can complete subtraction problems with minuends to 200, using number line.
3. Enrichment activities
a. Count-a-ladder.

G. Subtraction developed from the number line.
1. Prerequisite behaviors
a. Can find sums and missing addends for 2 or more addends using the number line.
2. Behaviors to be taught
a. Can find remainders or missing terms for subtraction exercises orally and written, using the number line.
b. Can show that subtraction is inverse of addition by completing equations.

        c. Finds sums and differences for horizontal and vertical problems, with missing term represented by a letter.
        d. Completes subtraction problems with minuends to 200 using the number line.
    3. Enrichment activities
        a. SRA drill tapes
        b. Count-a-ladder.

H. Elementary facts of subtraction
    1. Prerequisite behaviors (all stated in previous items)
    2. Behaviors to be taught
        a. Can tell orally how many are left using initial sets of 5 or less.
        b. Can supply subtraction facts for minuends to 12 with no pictured sets.
        c. Can subtract the set of numbers),1,2....N from a given number to create a family of related subtraction equations.
        d. Can find remainders or missing terms for subtraction equations to 12.
        e. Can find sums and differences for horizontal and vertical equations with missing term represented by letter.
        f. Can subtract without regrouping numbers to 200.

I. Subtraction, the inverse of adding a number (relation of addition and subtraction)
    1. Prerequisite behaviors (All stated in previous items)
    2. Behaviors to be taught
        a. Identifies number in subsets and altogether.
        b. Can find sums or missing addends.
        c. Can complete addition equation and related subtraction equation
    3. Enrichment activities
        a. Sum Fun game
        b. Sum Fun game on tape
        c. Count-a-ladder

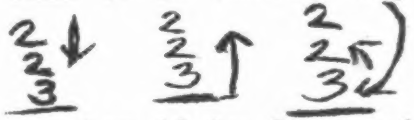
J. Property of one in subtraction
    1. Prerequisite behavior (listed previously)
    2. Behaviors to be taught
        a. Can show orderly sequence of subtracting one from a number.
        b. Can use 3 numbers to write or complete two addition equations, and their two related subtraction equations.
    3. Enrichment activities
        a. Count-a-ladder
        b. Plastic disc activities

K. Beyond tens (reading and/or writing words or numerals for numbers)
    1. Prerequisite behaviors are all previously stated.
    2. Behaviors to be taught
        a. Can write numerals as __hundreds, ___tens, ___ones (to 1000)
        b. Can write numeral for numbers stated as hundreds, tens, and ones.
           Ex. __________= 9 hundreds, 8 tens, 0 ones.

L. Multi-digit addition, no regrouping
    1. Prerequisite behaviors stated previously.
    2. Behaviors to be taught
        a. Can do problems with zero as addend.
        b. Can do problems in the tens, ending with zero. (as 20+30=50)
        c. Can do problems in the hundreds, ending in zero (100+210=310)

   - d. Can add two numbers horizontally or vertically without regrouping numbers to 200.
   - e. Can state and write numerals as hundreds, tens, and ones.

M. Addition with more than two addends, without regrouping.
 1. Prerequisite behaviors stated previously.
 2. Behaviors to be taught.
   - a. Adds three numbers less than 10 in various ways, showing associative property



   - b. Can add three numbers less than 10 by first adding numbers in parenthesis.
   - c. Identifies and completes problems showing commutative or associative properties of addition.
 3. Enrichment activities
   - a. Count-a-ladder
   - b. Cross Number puzzles (also on tape)

N. Multi-digit subtraction; no regrouping
 1. Prerequisites are previously stated
 2. Behaviors to be taught
   - a. Completes subtraction problems with minuends to 200 using algorithm.
   - b. Adds and subtracts numbers in tens and hundreds, ending in zero, and compares with elementary addition and subtraction.
   - c. Solves one step word problems adding and subtracting money values to 200.
   - d. Completes subtraction with minuends to 200 using a counting aid, such as abacus or number line.

O. Other names for a number
 1. Prerequisite behaviors stated previously
 2. Behaviors to be taught
   - a. Finds other names for numbers by writing simple addition expressions with or without pictured groups.
   - b. Finds other names by matching given addition expressions, with or without pictures.
   - c. Finds other names by matching given subtraction expressions, with pictures available.

P. Difference between number and numeral.
 1. Prerequisites listed previously
 2. Behaviors to be taught
   - a. Finds other names for numbers to 12 by writing or completing simple addition expressions.

Q. Associativity, a property of addition
 1. Prerequisites previously stated.
 2. Behaviors to be taught
   - a. Adds three numbers less than 10 in different order.
   - b. Adds three numbers less than 10 by first adding those in parentheses.
   - c. Completes problems showing commutative and associative properties.

R. Non-equivalent sets
1. Prerequisites stated previously
2. Behaviors to be taught
a. Pairs off objects in two sets.
b. Matches objects in sets, and responds more, less, or same.
c. Compares sets visually.
d. Places signs > or < between two numerals to indicate greater than, less than.

S. Zero, the identity element in addition
1. Prerequisites previously stated
2. Behaviors to be taught
a. Completes problems with zero as addend.
b. Identifies zero as identity element for addition.

T. Role of zero in subtraction
1. Prerequisites stated previously
2. Behaviors to be taught
a. Uses 3 numbers to write or complete two addition equations and related subtraction equations, showing inverse quality (0+5, 5+0, 5-0, 5-5)

U. One-to-many correspondence
1. Prerequisites stated previously
2. Behaviors to be taught
a. Uses equivalent measures of "2 cups= 1 pt.= 1 qt., 4 qts.= 1 gal. to solve problems requiring conversion.

V. Units of measure —money
1. Prerequisite behavior
a. Adds two numbers both in the tens, or one in hundreds and one in the tens for sums to 200.
2. Behaviors to be taught
a. Shows value of pennies, nickels, and dimes by matching pictures of coins with values in other coins or for making purchases.
b. Shows value of quarter by matching with its numerical value.
c. Shows value of a half dollar by matching with numerical value.
d. Totals collections of pennies, nickels, dimes, quarters, and half dollars to 99¢.
e. Selects coins to add to a given total for purchases.
f. Solves one step word problems involving adding or subtracting money values.
g. Adds or subtracts values of coins.
h. Indicates change to be received from making purchases.
3. Enrichment activities
a. Pay the cashier game
b. Cash register-play store activities.

III. Optional items which can be taught
A. Telling the temperature
B. Ordering numerals (skip counting, etc.)
C. Ordinal numbers
D. Regions of a plane surface (¼,½)

E. Regions, as in plane figures-rectangles, triangles, squares, circles.
F. Multiplication as repeated addition
G. Measurement (pounds, dozens, pairs)
H. Measurement (line segments)
I. Telling time to hour and half hour.
J. Quantity (dozen)
K. Fractional parts (subsets of a set)

IV. Enrichment Activities
    A. As listed with each item
    B. Geometric shape spotting game (4 players)
    C. Stories on tape
        1. Smallest Boy in the Class (size and measurement)
        2. The Wing on a Flea (geometric figures)
        3. Mrs. Popover Goes to Zoo (quantity)

Second Grade
Primes Math

- I. Items to be mastered
  - A. Equivalent sets (1-1 correspondence)
    - 1. Prerequisite behaviors
      - a. Identifies the correct number in a set
      - b. Identifies two matching or identical objects as forming a pair by drawing connecting lines.
    - 2. Behaviors to be taught
      - a. Matches objects in pictured sets by drawing connecting lines and responds that one set has more or less than the same number of a given set. Sets to 12.
      - b. Identifies the cardinal number of a structured set to 12 by writing a numeral.

  - B. Non-equivalent sets
    - 1. Prerequisite Behaviors
      - a. Same as those for equivalent sets
    - 2. Behaviors to be taught
      - a. Same as those for equivalent sets with emphasis on the non-equivalent

  - C. Develop the cardinal number zero
    - 1. Prerequisites
      - a. Identifies correct objects when asked to "Find the pictures of a set of" when a common property is named.
    - 2. Behaviors to be taught
      - a. Identifies a set that matches, draws or completes a set to match a given numeral, 0 to 12.
      - b. Identifies the empty set written with braces and the numerical value of the set.
      - c. Identifies the cardinal number of a structured group to 12.
      - d. Writes numerals 0 to 10 in forward or reverse counting order.

  - D. Reading and/or writing words or numerals for units, tens (10-99)
    - 1. Prerequisite behaviors
      - a. Write numerals 1-10 in forward or reverse counting order.
      - b. Matches a set with a numeral
      - c. Skip counts from any starting point 10's, 5's, 4's, 3's, 2's, 100's, and 50's forward and backward, responds orally.
    - 2. Behaviors taught
      - a. Identifies the place value of the units and tens digits in numbers 10-20, by drawing pictures of bundles of tens and ones.
      - b. Given dot pictures with one sequentially ordered number per dot, connects dots. (to 200)
      - c. Writes the missing numerals when presented with part of the sequence 1-200
      - d. Constructs and uses a number line to 200 to answer "what comes before" and "what comes after" or between two numbers.
      - e. Identifies cardinal number of a structured group to 200. Writes numeral.
      - f. Identifies the number of tens and ones in pictures representing numbers to 200 by writing or selecting the correct numeral.
      - g. Writes the numeral for a given number, to 99 as tens, units.
    - 3. Enrichment
      - a. 10x10 plastic peg board activity

II. Items to be introduced

A. Addition with more than two addends, without regrouping

1. Prerequisite behavior
    a. Identifies the number in each subset and the number altogether for sums up to 12.
    b. Add two numbers in horizontal or vertical notation form without carrying.
    c. Adds and subtracts numbers in the tens ending in zero and compares these operations with related elementary facts with numbers less than 10.
2. Behaviors to be taught
    a. Adds three numbers less than 10 in different ways showing the associative property of addition. Sums to 12
    b. Adds three numbers less than 10 by adding the numbers in the parentheses first. Sums to 12
    c. Identifies the number in each of a number of subsets and the number altogether, writes the numerals or responds orally. Numbers to 20
    d. Solves addition in vertical or horizontal form without pictures. Sums to 20.
3. Enrichment
    a. Game: Sum Fun

B. Ordering-greater, less, equal.

1. Prerequisite behaviors
    a. Place or between two numerals to indicate the greater or less for numbers to 12.
    b. Supplies the correct sign < or > to show the relationship between numbers to 200.
    c. Labels a shaded figure with appropriate fractional notation for any fraction ½, 1/3, to ¼.
2. Behaviors to be taught
    a. Provides correct sign , < or >= to show the relationship between whole objects divided into halves, thirds, fourths with one shaded part.
    b. Uses three feet in one yard to solve problems of comparison between inches, feet and yards.
    c. Identifies ½ inch as one of two equal parts making up one inch, 6 objects in ½ dozen; 6 inches as ½ of a foot.

C. Addition of fractions developed from regions

1. Prerequisite behaviors
    a. Identifies congruent regions and sub-regions.
    b. Lables a shaded figure with ½, 1/3, to ¼.
2. Behaviors to be taught
    a. Adding of shaded regions and associating fractional parts with the whole region using ½, 1/3 and ¼.
    b. Extend concepts to include ½.

D. Meaning of measurement (direct, indirect)

1. Prerequisite behaviors
    a. Constructs and/or uses a number line, 0 to 20, to state what number comes before or after a number or between two numbers. Extend number line to include numbers to 200.

2. Behaviors to be taught
  a. Identifies the number of units in the length of a line segment.
  b. Uses a ruler for measuring to the nearest inch.
  c. Identifies the units of liquid measure, cups, pints, quarts, half gallons and gallons.

E. Addition with more than two addends, with regrouping.
  1. Prerequisite behaviors
    a. Associative property of addition is reviewed.
    b. Solves addition equations in horizontal form sums to 20
    c. Add two or more addends using addition algorithm for regrouping.
  2. Behaviors to be taught
    a. Adds with regrouping to sums of 200 with two or more addends using a number line.
    b. Uses other names for numbers (with or without structured groups) to write alternate statements of a sum.
    c. Solves one-step word problems in addition.
    d. Uses addition algorithm to add three digit numbers with regrouping and sums to 1000.

F. Multiplication developed from union of two or more equivalent sets.
  1. Prerequisite behaviors
    a. Identifies the cardinal number of a structured group orally and by writing or selecting a numeral.
    b. Identifies the number in each of a number of subsets and the number there are altogether for sums up to 12.
  2. Behaviors to be taught
    a. When given pictures of two or more equivalent sets whose sum is 20 or less supplies orally the number altogether.
    b. Answers questions of the type "How many sets of each are equivalent to a set of ___" by partitioning sets. Products to 25.
    c. Commutative property of multiplication (Ex.___two's are ___or 3 sets of 2 are___.)
    d. Using the x sign to complete equations using sets of pictures.
    e. Solves problems represented by objects in arrays and completes multiplication equations with the x sign or repeated addition form. Products to 16.

G. Fractions developed from distances on the number line.
  1. Prerequisite behaviors
    a. Same as those used for meaning of measurement.
  2. Behaviors taught
    a. Compares fractional numbers using the number line; ½,¼,2/3,3/4.
    b. Locates fractional numbers including improper fractions and negative fractions on a number line. Numerators to 4.
    c. Measures a line segment to the nearest ½ inch, and ¼ inch.

H. Three-dimensional figures (solid)
  1. Prerequisite behaviors
    a. Responds correctly to directions which distinguish between a plane geometric shape and the region inside. Identifies, circular, rectangular, triangular, or square regions.
    b. Counts number of sides, right angles, or angles which a polygon has.
    c. Identifies a triangle, equilateral triangle, square, rectangle, circle and vertices of a polygon.

2. Behaviors taught
    - a. Selects objects having the same shape of distinguishing features from a collection of objects or pictured objects.

I. Describe and analyze:point
- 1. Prerequisite behaviors
    - a. Points on a number line identified.
- 2. Behaviors taught
    - a. Identifies a line as a set of points. (Taught as a part of lines, segments, and rays)

J. Describe and analyze: angles
- 1. Prerequisite behavior
    - a. Identifies a ray as a figure formed by one fixed end point and a line extending indefinitely in one direction.
- 2. Behaviors taught
    - a. Identifies an angle as the figure formed by two rays with a common endpoint.
    - b. Count the number of sides, right angles, or angles which a polygon has. sides to 6.
    - c. Identifies a right angle as the angle forming a square corner or as an angle which has 90°.

K. Describe and analyze: line, line segment, rays.
- 1. Prerequisite behaviors
    - a. Identifies a triangle, equilateral triangle, square, rectangle, pentagon, circle and vertices of a polygon.
- 2. Behaviors taught
    - a. Identifies a line as a set of points.
    - b. Identifies a line segment as finite in length and part of a line.
    - c. Identifies a line segment, endpoints, a ray as a figure formed by one fixed endpoint and a line extending indefintely in one direction.
    - d. Identifies a representation for a line as a straight line with arrowheads on each end, showing that the line goes on without stopping.
    - e. Identifies an angle as the figure formed by two rays with a common endpoint.
    - f. Names a line by letters of any two points on it.

L. Multi-digit substraction-regrouping
- 1. Prerequisite behaviors
    - a. Adds two numbers both in the tens, or one in the hundreds and one in the tens for sums to 200 using expanded notation.
    - b. Adds two or more addends (to 3 digits) using the addition algorithm for regrouping.
    - c. Completes subtraction problems with minuends to 200 by using structured groups.
    - d. Subtracts numbers in horizontal or vertical form without regrouping. Numbers to 200.
- 2. Behaviors to be taught
    - a. Completes subtraction problems with minuends to 200 by using a number line.
    - b. Performs repeated subtraction with minuends to 200 and subtrahends 10 or any multiple of 10.
    - c. Subtracts using expanded notation and regrouping, also using the subtraction algorithm involving numbers to 200.
    - d. Performs subtraction for problems with numbers to 3 digits using above method.

M. Commutatively, a property of multiplication (whole numbers)
1. Prerequisite behaviors
a. Illustrates the use of commutative property of addition by writing two different addition equations for the same pictured situation. (to 12)
b. Completes 2 multiplication equations written as "___two's are ___" or 3 sets of 2 are ___" for the same set of pictured objects which show the commutative property. Products to 1[illegible].
2. Behaviors to be taught
a. Uses pictured arrays to illustrate and/or solve simple problems involving the commutative property for multiplication.
b. Completes two multiplication equations written with the x sign which are represented by the same set of pictured objects and together show use of the commutative property for multiplication. Products to 1[illegible]. (with or without pictures)

N. Division-developed from partitioning into equivalent sets.
1. Prerequisite behaviors
a. When given pictures of two or more equivalent sets whose sum is 20 or less, supplies orally the number altogether when there are____ sets of ___each.
b. Answers questions of the type: "How many sets of __each are equivalent to a set of____" by partitioning sets. For products to 25.
2. Behaviors to be taught
a. Divides as set of up to 1[illegible] members into a requested number of equivalent sets and states the number in each of the sets and the number of remaining objects, if any.
b. Writes the missing factor and quotient for a pictured array showing that multiplication and division are inverses. Products to 20.
c. Demonstrates with objects or pictured arrays that a/b=x means that we can divide a to make x sets with be in each set or that if we have b sets there will be x things in each set.

O. Division-inverse of multiplication
1. Prerequisite behaviors
a. Same as above for equivalent sets in division.
b. Divides a set of up to 1[illegible] members into a requested number in each of the sets and the number of remaining objects, if any.
2. Behaviors to be taught
a. Writes or completes the related division problem for a multiplication problem with product 1[illegible] or less.

P. Multiplication developed from the number line.
1. Prerequisite behaviors
a. Find the sum of two or more addends by using the number line
b. Adds with regrouping to sums of 200 with two or more addends using a number line.
c. Writes the answers when both the repeated addition form and the multiplication form written with the x sign are presented. Products to 1
2. Behaviors to be taught
a. Uses a number line monogram to solve multiplication problem for facts through 10x10.
b. Solves problems (multiplication) for products to 200 using the number line.

Q. Multi-digit addition two addends with regrouping.
1. Prerequisite behaviors
a. Writes a numeral as hundreds, tens, ones.
b. Adds two numbers in horizontal or vertical notation form without carrying.
2. Behaviors to be taught
a. Adds with regrouping to sums of 200 with two or more addends using a number line.
b. Adds all numbers in the tens to sums of 100 (with regrouping) using the addition algorithm.
c. Adds two or more addends (to 3 digits) using the addition algorithm for regrouping.

R. Rounding Numbers
1. Prerequisite behaviors
a. Taught previously to this
2. Behaviors to be taught
a. Finding the nearest numeral, ten using the number line.
b. Using the number line to find the nearest hundred.

S. Historical systems of numeration
1. Prerequisite behaviors
a. Writes numerals, identifies sets, identifies place value of the units and tens digit in numbers 10 to 20.
b. Identifies cardinal number of a structured group.
2. Behaviors to be taught
a. Identify and write Roman numerals associated with Arabic numerals.
b. Identifies the Egyptian and Babylonian numeral systems orally.
c. Associated Roman Numerals with the Arabic numerals on the clock face.
d. Identifies book volumes labeled with Roman numerals.

III. Optional items
A. Ordinal numbers first to twentieth
1. Calendar - through thirty first
a. days, months

B. Measure
1. Time - hour, half hour, five minute intervals - quarter to, and quarter after.

C. Money
1. Nickel, dime, quarter, half dollar and dollar,

D. Secret code puzzles - addition and subtraction drills.

E. Cross Number Puzzles

F. Open sentences in all number operations.

IV. Enrichment activities
A. A file of additional activities for enrichment is available. Games, directed mathematics skills, and activities are suggested.

Third Grade
Primes Math

- I. Items to be mastered
  - A. Elementary facts of addition
    - 1. Prerequisites
      - a. Illustrates the use of the commutative property of addition by writing two different addition equations for the same pictured situation.
      - b. Finds the sum of two or more addends by using the number line.
      - c. Completes problems in which zero is one of the addends.
    - 2. Behaviors to be taught
      - a. Finds other names for numbers to 20 by matching given addition expressions.
      - b. Finds other names for numbers to 20 by writing or completing simple addition expressions.
      - c. Solves one-step word problems with pictures which require the use of addition facts with numbers to 20.
      - d. Solves addition porblems written in vertical notation form without pictures. Sums to 20
      - e. Constructs and/or completes an addition table and uses the table to solve addition problems.
    - 3. Enrichment
      - a. Tape-Riddles
  - B. Cardinal numbers beyond ten
    - 1. Prerequisites
      - a. Adds to a second set or draws a second set of objects and identifies one set as having more, fewer, or the same number of objects as the other set.
      - b. Identifies the cardinal number of a structured group. Responds by writing or selecting a numeral.
      - c. Writes the proper numeral when shown a set written in notation followed by an equals sign.
    - 2. Behaviors to be taught
      - a. Identifies the number of tens and ones in pictures representing numbers to 200 by writing or selecting the correct numeral.
      - b. Renames or regroups numbers according to the place value of the digits - e.g., 275 = 1 hundred, __tens, 5 ones, or 2 hundreds, 5 tens, ___ones.
      - c. Abstracts the cardinal number from structured groups to 1000 in number. Writes the standard numeral. Objects are grouped into bundles of hundreds, tens, ones.
  - C. Ordinal counting-sequence
    - 1. Prerequisites
      - a. Identifies how many more or less one set has when given two nonequivalent sets.
      - b. Supplies the correct sign < or > to show the relationship between two numbers to 200.
      - c. Constructs and uses a number line to 200 to answer questions, such as, what number comes before, after, or between two numbers.

2. Behaviors to be taught
    a. Locates the requested ordinal position, first to twelfth, in an ordered set of objects.
    b. Writes the numeral that comes after, before, or between two given numbers to 1,000 to 10,000.
    c. Writes the missing numerals when presented with part of the sequence of numbers from 1 to 10,000 with blanks or frames for missing numbers.
    d. Writes the correct numeral for a number given in words up to millions.
    e. Reads numbers to millions.

D. Multi-digit addition-no regrouping
  1. Prerequisites
    a. Identifies or completes problems which illustrate the commutative or associative property of addition.
    b. Rewrites a given problem to illustrate the commutative or associative property of addition.
    c. Writes a numeral in expanded notation form; its "hundreds" part, its "tens" part, and its "ones" part. Renames numbers to 1,000.
  2. Behaviors to be taught
    a. Adds two numbers in horizontal or vertical form without carrying. Sums to 200.
    b. Supplies or completes an addition and subtraction equation for word problems. Solves the equation.
    c. Adds using cent notation and decimal notation for money problems to $2.00.
    d. Adds two, three or more addends (to five digits) for problems written in horizontal or vertical notation form.
    e. Adds or subtracts money values using cent notation or decimal notation, no limit to money value.
  3. Enrichment
    a. Tape-find the hidden picture

E. Backward-rote counting
  1. Prerequisites
    a. Abstracts the cardinal number from structured groups to 1000 in number. Writes the standard numeral. Objects are grouped into bundles of hundreds, tens, ones.
    b. Writes the missing numerals when presented with part of the sequence of numbers from 1 to 10,000 with blanks or frames for missing numbers.
  2. Behaviors to be taught
    a. Writes the numeral for a number that comes after, before, or between two given numbers to 1,000 to 10,000.
    b. Constructs and uses a number line to 200 to answer questions, such as, what number comes before, after, or between two numbers.

F. Addition developed from union of disjoint set or joining action
  1. Prerequisites
    a. Identifies the cardinal number of a structured group to 12. Responds by writing or selecting a numeral.
    b. Abstracts the cardinal number from structured groups at 1000 in number. Writes the number as —tens. Objects are grouped into bundles of hundreds, tens, ones.

2. Behaviors to be taught
    a. Supplies the missing operational sign, +,-,to complete problems for numbers to 100.
    b. Writes an addition equation using the + sign or a subtraction equation using the - sign to show that the given pictures depict an addition or subtraction equation.
    c. Identifies or completes problems which illustrate the commutative or associative property of addition.

G. Addition developed from the number line
  1. Prerequisites
    a. Supplies the correct sign < or > to show the relationship between two numbers.
    b. Skip counts from any starting point forward or backward by 10's, 5's, 4's, 3's, 2's, 100's, and 50's. Responds by writing numerals to continue the pattern.
  2. Behaviors to be taught
    a. Finds the sum of two or more addends by using the number line.
    b. Supplies the missing addend for problems with up to three addends.
    c. Adds with regrouping to sums of 200 with two or more addends using a number line.

H. Subtraction developed in relation to subsets or separating action.
  1. Prerequisites
    a. Identifies a subset of a given set by choosing among a number of sets, one or more which would qualify as subsets of the given set.
    b. Identifies how many more or less one set has when given two nonequivalent sets.
    c. Matches objects in pictured sets by drawing connecting lines and responds that one set has more, has less, or has the same number as a given set.
  2. Behaviors to be taught
    a. Takes away enough members of one set to match the numerical value of a given set. Answers: "How many are left?" and "How many were in the first set before?"
    b. Finds the remainder or a missing term for subtraction exercises.
    c. Provides or completes a subtraction statement using the - sign and = sign for pictures composed of sets of objects. Responds by writing or selecting the numerals.
  3. Enrichment
    a. Riddles - General review - no multiplication or division.

I. Subtraction developed from the number line
  1. Prerequisites
    a. Supplies the missing addend for problems with up to three addends and sums to 25.
    b. Skip counts from any starting point forward or backwards by 10's, 4's, 3's, 2's, 100's, and 50's. Responds by writing numerals to continue the pattern.
  2. Behaviors to be taught
    a. Completes subtraction problems with minuends to 200 by using a number line.
    b. Supplies the missing addend for problems with up to three addends and sums to 25.
    c. Supplies or completes the inverse problem in subtraction for a given addition problem.

- J. Multi-digit addition with regrouping
  - 1. Prerequisites
    - a. Rewrites a given problem to illustrate the commutative or associative property.
    - b. Writes a numeral in expanded notation form, Its "hundreds" part, its "tens" part and its "ones" part (renaming numbers). Numbers to 1000
    - c. Supplies an example of a word problem when requested to illustrate addition.
  - 2. Behaviors to be taught
    - a. Adds two or more addends (to five digits) using the addition algorithm for regrouping.
    - b. Solves for the missing addend for problems with numbers to five digits and requiring regrouping.
    - c. Solves one-step word problems in addition.
    - d. Checks answers in addition by applying the commutative property.
  - 3. Enrichment
    - a. Find the hidden picture.

- II. Items to be introduced
  - A. Number patterns - Rules for divisibility
    - 1. Prerequisites
      - a. Solves for the missing addend for problems with numbers to five digits and requiring regrouping. A placeholder may be used for one or more missing digits.
      - b. Subtracts using expanded notation and regrouping for problems involving numbers to 200.
      - c. Finds the missing factor (divisor, quotient) or dividend in division problems with or without placeholders. Numbers to 200.
    - 2. Behaviors to be taught
      - a. Identifies odd and even numbers from 0 to 200 by selecting or writing the numerals or responding orally.
      - b. Writes numerals to continue an addition sequence after discovering the rule intuitively.
      - c. Constructs and uses number families from basic addition facts. Sums to 24.
      - d. Writes numbers to continue a subtraction sequence after discovering the rule intuitively.
      - e. Completes "Cross number puzzles" involving subtraction for minuends to 200 (e.g. the difference of the differences of the rows in equal to the differences of the differences of the columns).
    - 3. Enrichment
      - a. Grids - Fill in squares so that all columns and rows add up to a given number.

  - B. Arithmetic Progressions
    - 1. Prerequisites
      - a. Adds with regrouping to sums to 200 with two or more addends using the addition algorithm.
      - b. Completes subtraction problems with minuends to 200 by using the subtraction algorithm for regrouping.
    - 2. Behaviors to be taught
      - a. See number patterns

  - C. Common fraction notation - equal denominators
    - 1. Prerequisites
      - a. Writes the number of equivalent subsets there are when a group of objects i divided into halves, thirds, fourths, to twelfths.

C. Common fraction notation - equal denominators
1. Prerequisites
a. Writes the number of equivalent subsets there are when a group of objects is divided into halves, thirds, fourths, to twelfths.
2. Behaviors to be taught
a. Identifies ½, 1/3, or ¼ of regular whole objects by selecting or writing the fraction which describes the shaded object.
b. Uses objects, pictured objects, or no pictured objects to identify different fractional statements which are equivalent to each other.

D. Equivalent common fractions
1. Prerequisites
a. Writes the number of parts there are when an object is divided into halves, thirds, fourths, to twelfths.
b. Writes the number of equivalent subsets there are when a group of objects is divided into halves, thirds, fourths to twelfths.
2. Behaviors to be taught
a. Behaviors to be taught provides the correct sign >, < or = to show the relationship between whole objects divided into halves, thirds, fourths, thirds with one part shaded.
b. Uses objects, pictured objects, or no pictured objects to identify different fractional statements which are equivalent to each other.

E. Addition of fractions - developed from the number line
1. Prerequisite
Uses the number line to identify different fractional statements which have the same numerical value. Fractions to twelfths with numerators to 4.
2. Behavior to be taught
Locates fractional numbers including improper fractions and negative fractions on a number line. Numerators to four.

F. Use of commas to separate into periods
1. Prerequisites
a. Writes the standard numeral for a number represented as ____thousands, _____hundreds, _____tens, ____ones. Numbers to 10,000.
b. Renames or regroups numbers according to the place value of the digits, e.g. 275=2 hundreds, 7 tens, 5 ones; or 1 hundred ____tens, 5 ones; or 2 hundreds, 5 tens, ___ones.
c. Writes the standard numeral for a number represented as ___hundreds, _____tens, ____ones. Numbers to 1000.
2. Behaviors to be taught
a. Writes a number in expanded notation form; its "thousands" part + its "tens" part + its "ones" part. Numbers to 10,000.
b. Reads numbers to millions
c. Identifies the place value of digits in numbers up to millions.

G. Multiplication developed from arrays
1. Prerequisite
a. Identifies the correct objects in a picture when asked to "Find the picture of a set of ___" when a common property is named.
2. Behaviors to be taught
a. Solves multiplication problems represented by objects in arrays and completes multiplication equations written with the x sign or in repeated addition form.

b. Solves a more difficult multiplication problem by using the results of two smaller arrays to obtain the product of the larger array.
c. Checks multiplication problems by reversing the order of the factors and multiplying.
d. Constructs or completes a multiplication table through the product 12x12.

H. Property of zero in multiplication

1. Prerequisites
   a. Uses pictured arrays to illustrate and/or solve simple problems involving the commutative property for multiplication. Products to 18.
   b. Writes the answers when both the repeated addition form and the multiplicati form written with the x sign are presented.
2. Behaviors to be taught
   a. Uses a number line nomogram to solve multiplication problems for facts through 10x10.
   b. Gives zero as the product for a multiplication problem for which zero is one of the factors.

I. Multi-digit multiplication without regrouping

1. Prerequisites
   a. Identifies one as the identity element for multiplication.
   b. Multiplies more than two numbers using the associative principle for multiplication.
   c. Identifies or selects from a list those number statements which illustrate a certain principle, e.g., associative commutative, distributive, etc.
2. Behaviors to be taught
   a. Uses the distributive principle to simplify multiplication of one and two digit factors, e.g., 7x85 = (7x80) + (7x5).
   b. Multiplies a one digit factor times a three or more digit factor.
   c. Solves one or two-step word problems in multiplication.
3. Enrichment
   a. Multiplication puzzle

J. Multi-digit multiplication with regrouping

1. Prerequisites
   a. Uses the associative principle to simplify multiplication of one and two digit factors; e.g., 2x (98x50) = (2x50) x 98.
2. Behaviors to be taught
   a. Solves multiplication problems for facts 10 x 10 using the multiplication algorithm for problems written in vertical or horizontal form.
   b. Multiplies a one digit factor times a three or more digit factor. Uses algorithm with or without regrouping in tens, hundreds, or thousands place.

K. Multiplication - more than two factors.

1. Prerequisites
   a. Solves multiplication problems by using other names for numbers (renaming) and the distributive property.
2. Behaviors to be taught
   a. Multiplies more than two numbers using the associative principle for multiplication.
   b. Uses the associative principle to simplify multiplication of one and two digit factors.

L. Multiplier or powers of ten as factors

1. Prerequisites
   a. Completes two multiplication equations which together illustrate use of the commutative property for multiplication. Pictures may or may not be presented with the problem.

2. Behaviors to be taught
    a. Uses multiples of ten to generalize multiplication and division facts, e.g., 3x9=___, 3x90=___, and 3x900=___.
    b. Multiplies using the algorithm for multiples of powers of 10, e.g. 8x10=80, 8x20=160, 8x100=800.
    c. Uses the division algorithm to solve problems for multiples of powers of 10, $100 \div 4$, $20 \div 4$.

M. Distributivity of division over addition
  1. Prerequisites
    a. Divides a set of up to 18 members into a requested number of equivalent sets and states the number in each of the sets and the number of remaining objects, if any.
    b. Supplies or completes an inverse problem in division for a given multiplicati problem; with or without remainders.
    c. Supplies or completes inverse problem in multiplication for a given division problem.
  2. Behaviors to be taught
    a. Illustrates that division is distributive over addition and subtraction when given a dividend and a divisor.
    b. Uses the distributive principle to simplify division problems for two or three digit products; e.g. $248 \div 8 = (240+8) \div 8 = (240 \div 8) + (8 \div 8)$
    c. Varies the placement of parenthesis in a problem to generate different problems and then solves each.
    d. Identifies or selects from a list those number statements which illustrate a certain principle; e.g. Associative, commutative, distributive, etc.

N. Elementary facts of division
  1. Prerequisites
    a. Solves division problems by performing repeated subtractions.
    b. Writes or completes the related division problem for a multiplication problem with product 18 or less.
  2. Behaviors to be taught
    a. Writes division equations when presented with written stories.
    b. Solves division problems for dividends to 100 with whole number quotients by using the division algorithm.

O. Division - divisor less than ten, dividend not regrouped, no remainder.
  1. Prerequisites
    a. Identifies one as the identity element for division.
    b. Checks answers in addition, subtraction, multiplication, division by using the inverse operation.
  2. Behaviors to be taught
    a. Divides money values by repeatedly subtracting multiples of the divisor, using cent notation or decimal notation to dividends to 200; with or without remainders.
    b. Divides a three or more digit product by a one digit factor by using the algorithm with regrouping or without regrouping. Checks division problem by multiplication.

III. Optional
  A. Multi-digit subtraction regrouping (This is an important item although not mastered or introduced at this grade level).
  B. Ability to round numbers for estimates.
  C. Money
  D. Time
  E. Temperature
  F. Tables, charts, graphs.
  G. Describe and analyze geometric figures.

Grade Four
Primes Math

I. Items to be mastered
  A. Elementary facts of subtraction
    1. Prerequisites
      a. Constructs and completes an addition and uses the table.
      b. Solves addition problems.
      c. Fills in frames for missing addend problems.
      d. Completes the inverse problem in addition.
    2. Behaviors to be taught
      a. Using the number line, finds the remainder of a missing term for subtraction.
      b. Completes addition and related subtraction equations.
      c. Finds the missing terms for a collection of addition and subtraction problems.
    3. Enrichment
      a. Enrichment — dots and squares

  B. Multiplication developed from equivalent sets
    1. Prerequisites
      a. Abstracts the cardinal number from pictures of structed groups to five in number. Responds by writing a numeral.
      b. Matches objects in pictured sets by drawing connecting lines and responds that one set has more, has less, or had the same number as a given set. Sets to five.
      c. Identifies a subject of a given set by choosing among a number of sets, one or more which would qualify as subsets of the given set.
    2. Behaviors to be taught
      a. Answers questions of type "How many set of each are equivalent to set of M.
      b. Completes 2 multiplication equations written as twos are on 3 sets of 2 are for the same set of pictured objects which show the use of the commutative property. Products to 18.
      c. Completes two multiplication equations written with X sign which are represented by the same set of pictured objects and together show use of the commutative property for multiplication. Products to 18.
      d. Completes or writes problems written in terms such as twos are 3 sets of 2 are, with the word times, in equation form with the x sign or in repeated addition form. Products to 200.
    3. Enrichment
      a. None

  C. Multi-digit subtraction no regrouping
    1. Prerequisite
      a. All subtraction combinations
      b. Completes cross no. puzzles involving addition to sums to 200.
      c. Rounds numbers to nearest thousand, hundred, thousand or million for estimation of answers for problems.
    2. Behaviors to be taught
      a. Performs subtraction with numbers to 5 digits and not requiring regrouping.
      b. Constructs and uses number families for solutions in subtraction to 10,000.

D. Addition with more than two addends with regrouping
   1. Prerequisite
      a. Rewrites a given problem to illustrate this commutative or associative property of addition.
      b. Adds two, three or more addends — no regrouping
      c. Adds two, three or more addends with regrouping.
      d. Adds 3 or more addends using the commutative principle or associative principle.
   2. Behaviors to be taught
      a. Adds or subtracts money values.
      b. Solves one step —word problems involving combination of any arithmetic operations.
   3. Enrichment
      a. Cross number puzzle.

E. Commutativity, a property of
   1. Prerequisite
      a. Finds the missing terms for a collection of addition and subtraction problems written in horizontal or vertical form. Sums to 20. Missing term maybe represented by a letter.
      b. Constructs and/or completes an addition table.
   2. Behaviors to be taught
      a. Identifies or completes problems which illustrate the commutative or associative property of addition.
   3. Enrichment
      a. None

F. Multiplication developed from number line.
   1. Prerequisite
      a. Skip counting
      b. Adds two numbers both in the tens
      c. Adds 200 more addends to 5 digits using the addeteou algorithm for regroupin
   2. Behaviors to be taught
      a. Solves multiplication problems represented by object in arrays and completes multiplication equations written with the x sign or in repeated addition form. Products to 200.
      b. Solves multiplication problems through products to 200 using repeated addition.

G. Subtraction — Inverse of addition
   1. Prerequisites
      a. Adds and subtracts numbers in the hundreds ending in zero, with or without structured groups and compares these operations with the related elementary fact with numbers less than 10.
      b. Does problem relating to the closure property of addition.
      c. Writes numbers to continue a subtraction sequence after discovering the rule intuitively.
   2. Behaviors to be taught
      a. Supplies or completes the inverse problem in addition for a given subtraction problem.
      b. Supplies or completes the inverse problem in subtraction for a given addition problem.
      c. Checks subtraction problems by the difference and subtrahend to equal the minuend.

H. Multi-digit subtraction with regrouping
  1. Prerequisites
    a. Performs subtraction for problems with numbers to 5 digits — no regrouping
    b. Constructs and use number families for solutions in subtraction minuends to 10,000.
    c. Writes numerals in expanded form.
    d. Adds two numbers both in tens, or one in hundreds and one in tens for sums to 200 (regrouping or renaming)
  2. Behaviors to be taught
    a. Using number line to complete subtraction with minuends to 200.
    b. Subtracts using cent notation or decimal notation for money problems.
    c. Subtraction for problems of 5 digits — (regrouping)

I. Multiplication developed as repeated addition
  1. Prerequisites
    a. Behaviorial analysis is progress
  2. Behaviors to be taught
    a. Solves multiplication problems through products to 200 using repeated addition.
    b. Using arrays to solve multiplication problems.
  3. Enrichment
    a. Game — Build you Thinking Power.

J. Elementary facts of multiplication
  1. Prerequisites
    a. Answers questions of the type "how many sets of each are equivalent to a set of M" by drawing pictures of sets or arrays.
    b. Solves multiplication problems through products to 200 — using repeated addition.
    c. Completes 2 multiplication equations which together illustrate use of commutative property for multiplication. Products to 100.
  2. Behaviors to be taught
    a. Checks multiplication problems by reversing the order of the factors and multiplying.
    b. Constructs or completes a multiplication table through the product 12x12.
    c. Solves multiplication problems by using other names for numbers (renaming) and the distributive property. Products to 200.
  3. Enrichment
    a. Puzzle

K. More than 2 factors — no regrouping
  1. Prerequisites
    a. Completes 2 multiplication equations which together illustrate use of the commutative property for multiplication.
    b. Identifies the product and/or factors of a problem. Products to 200.
  2. Behaviors to be taught
    a. Constructs and completed a multiplication through the product of 2x12.

L. Elementary facts of division
  1. Prerequisites
    a. Writes or completes the related division problems for a multiplication problem with product 18 or less.
    b. Checks multiplication problems by dividing the product by one of the factors to get the other factor.

2. Behaviors to be taught
    a. Constructs a rectangular array to illustrate a given divisor statement.
    b. Writes or completes the related division problem for a multiplication problem with product 18 or less.
    c. Solves multiplication and division problems with small whole numbers solutions with placeholders for missing numbers by performing the inverse operation to the one indicated. Numbers to 200.

M. Division developed from equivalent sets.
1. Prerequisites
    a. Matches objects in pictured sets by drawing connecting lines and responds that one set is more, has less, or has same number as a given set.
    b. Answers questions of the type how many sets of each are equivalent to a set of M.
    c. Identifies a subset of a given set by choosing among a number of sets, one or more which would qualify as subsets of the given set.
2. Behaviors to be taught
    a. Solves division problems by repeately subtracting multiples of the divisor. Dividends to 200 with remainders.
    b. Solves division problems by performing subtraction of the sets.

N. Divisors less than 10 - no regrouping no remainder
1. Prerequisites
    a. Solves multiplication problems for facts through 10x10 - using the multiplication algorithm for problems written vertical or horizontal.
    b. Multiplies a one digit factor times a 3 or more digit factor.
    c. Uses the distributive principle to simply multiplication of one or 2 digit factors.
2. Behaviors to be taught
    a. Find the missing factor in multiplication with missing factors to 10 and products to 100.
    b. Finds the missing factor or dividend in division facts, 5x9=3x90 = 3x90.
    c. Divides a 3 or more digit product by a one digit factor by using the algorithm.

O. Definition: set of whole numbers
1. No prerequisites
2. Behaviors to be taught
    a. Identifies the cardinal number of a structured group to 12.
    b. Identifies a set of whole numbers as a set beginning with zero as the smallest number and having no greatest whole number.
3. Enrichment: Using a code

II. Items to be introduced

A. Whole numbers related to a set of fractions
1. Prerequisites
    a. Shows by example that the numerator of a fraction means that many out of a number of equal parts expressed by the denominator for the fractions, 2/2, 2/3, 2/4.
    b. Identifies other names for numbers by choosing from a selection of proper fractions, improper fractions, mixed fractions.

2. Behaviors to be taught
   a. Supplies correct sign > < or = to show comparisons of fractions.
   b. Uses objects, pictured objects to identify different fractional statements which are equivalent to each other.
   c. Finds a set of equivalent fractions for a given fraction using higher and lower terms.

B. Closure, a property of addition
   1. Prerequisites
      a. Does problems relating to closure of addition
   2. Behaviors to be taught
      a. Identifies which operations or number system (whole numbers) have the closure property.
   3. Enrichment - Writing numerals to hundred thousands.

C. Subtraction - Non closure, non commutative, non associative.
   1. Prerequisites
      a. Shows by performing the operation that subtraction is not commutative for interchange of the positive numbers, even though addition is.
      b. Rewrites a given problem to illustrate the commutative or associative property of addition.
   2. Behaviors to be taught
      a. Identifies which operations (whole numbers, numbers have the closure property)
      b. Does problems showing that subtraction is not associative.

D. Subtraction is a binary operation
   1. Prerequisites
      a. Fills in frames for missing addend problems with two addends.
      b. Supplies or completes the inverse problem in subtraction for a given addition problem.
   2. Behaviors to be taught
      a. No new ones.

E. Division with ten or greater
   1. Prerequisites
      a. Review all elementary division facts.
      b. Divide a 2 or more digit product by a 2 or 3 digit factor using the Algorithm. Checks on division problems by multiplication.
      c. Prefer addition, subtraction, multiplication or division - using numbers to 1000, 10,000, 100,000.
   2. Behaviors to be taught
      a. Illustrate that divisor is distributive over addition and subtraction when given a dividend or divisor.
      b. Divide a two or more digit factor by using the Algorithm.
      c. Solves more problems involving addition, subtraction, multiplication or division using numbers to 1,000, 10,000, 1,000,000.

F. Average
   1. Prerequisites
      a. Solves one step word problems involving combination of any arithmetic operation.
      b. Solves division word problems to 200.
      c. Estimates answers in addition for sum of 200 by rounding numbers to get a simple form.

2. Behaviors to be taught
    a. Finds average of 2 or more numbers in numerical problem form.
    b. Develops the rule for finding an average using simple problem situations.
    c. Solves word problems requiring the finding of simple averages for quotients to 10.
3. Enrichment
    a. Find radius and diameter.

G. Functions and relations
1. Prerequisites
    a. Functions and relations are developed simultaneously with and/as an advanced activity in basic operations, sense of cardianalety, number of theory, and mathematical structure.
    b. Its prerequisites include any mathematical operations or relationships being investigated.
2. Behaviors to be taught
    a. Finds the missing factor or product for multiplication problems with placeholders including letters. Uses or draws an array to find the missing factor or product. Products to 200.

H. Divisor less than 10, regrouping remainder
1. Prerequisites
    a. Solves division problems by repeadtedly subtracting multiple of divisor.
    b. Uses the distributive principle to simplify division problems for 2 or 3 digit products.
2. Behaviors to be taught
    a. Divisors and quotients to be taught
    b. Divides a three or more digit product by a one digit factor.
    c. Checks division by multiplication.
    d. Solve equations with literal values which necessitate finding the greatest quotient and appropriate remainder.
3. Enrichment (circles)

I. Division - non closure - non commutative, non associative
1. Prerequisites
    a. Uses the commutative principle for multiplication to solve problems with a 2 place factor.
    b. Uses the associative principle in multiplying a 3 or more digit factor by 1,2,3 factors.
    c. Identifies or selects from a list those number statements which illustrate a certain principle.
2. Behaviors to be taught
    a. Illustrates that the commutative principle does not hold for division using small whole numbers.
3. Enrichment (none)

III. Optional Items
A. Roman numerals
B. Amount of Money
C. Measurement, temperature, liquids, weight
D. Finding perimeter
E. Multiplication - multiplies 2 digits
F. Graphs, line graph
G. Geometry
H. Fractional numbers

Grade Five
Primes Math

I. Items to be mastered
  A. Multi-digit multiplication — two factors — regrouping
    1. Prerequisite behavior
      a. Uses multiplication algorithm for problems up to 10x10.
      b. Multiplies a one digit factor times a three or more digit factor.
      c. Uses distributive principle to simplify multiplication of one and two digit factors.
    2. Behaviors to be taught
      a. Multiply using lattice method.
      b. Multiplication by renaming — to 200.
      c. Multiply using algorithm for multiples of powers of 10. e.g. 8x10-80. Also 3 digit numbers times 3 or more digit numbers.

  B. Two combined operations
    1. Prerequisites
      a. Adds two or more addends
      b. Performs subtraction problems to five digits.
      c. Solves word problems using addition, subtraction, multiplication or division.
      d. Divides — check by multiplication
    2. Behaviors to be taught
      a. Solves multi-step word problems.
      b. Finds truth set for open sentences.
      c. Makes truth set.

  C. Division — Divisor less than 10, dividend regrouped, remainder.
    1. Prerequisites
      a. Solves division problems for dividends to 100.
      b. Writes division remainder as a fraction.
      c. Uses distributive principle to simplify division problems.
      d. Supplies difference between a number and a product expressed as two factors.
    2. Behaviors to be taught
      a. Solves division problems by subtracting multiples of divisor.
      b. Solves word problems to dividends of 200.
      c. Behavior analysis in progress.

  D. Division inverse of multiplication
    1. Prerequisites
      a. Solves multiplication problems to 200 using number line.
      b. Solves division problems by performing repeated subtractions.
    2. Behaviors to be taught
      a. Solves multiplication and division numbers to 200.
      b. Division by zero is undefined.
      c. Writes the fractional equivalent for a division problem.

  E. More than two factors regrouping
    1. Prerequisites
      a. Understands multiplication of zero.
      b. Multiplies a one digit factor times a three or more digit factor.
    2. Behaviors to be taught
      a. Multiplies using associative principle for multiplication. Products to 200.
      b. Simplifies multiplication using associative principle.
    3. Enrichment
      a. None

F. Division: divisor less than ten, dividend not regrouped remainder.
   1. Prerequisites
      a. Solves division problems by performing repeated subtractions.
      b. Writes remainder of division problem as a fraction.
   2. Behaviors to be taught
      a. Behavioral analysis in progress.
   3. Enrichment
      a. None

G. Multiplication = multiples of ten as a factor
   1. Prerequisites
      a. Multiplies using zero
      b. Can multiply a one digit factor times a three or more digit factor.
   2. Behaviors to be taught
      a. Uses multiples of ten to generalize multiplication and division facts.
      b. Uses associative and distributive principle to simplify multiplication.
      c. Rounds numbers to numbers between thousand —million.
   3. Enrichment
      a. None

H. Commutativity, a property of addition for fractions
   1. Prerequisites
      a. Subtraction is not the commutative or associative property of addition.
      b. Adds fractions to sums of 4/4 using picture aids.
   2. Behaviors to be taught
      a. Uses closure property in adding fractions.
      b. Uses commutative and associative principles in adding fractions.
      c. Adds numbers with one or two decimal places.
   3. Enrichment
      a. None

I. Common fraction notation — equal denominators.
   1. Prerequisites
      a. Adds or subtracts any two fractions with equal denominators.
      b. Behavior analysis in progress.
   2. Behaviors to be taught
      a. Adds fractions with equal denominators.
      b. Adds or subtracts mixed or improper fractions.
      c. Adds two numbers with whole number parts and one or two decimal places.
      d. One step word problems involving any fractions to twelfths.
   3. Enrichment
      a. None

J. Division: divisor less than ten, dividend regrouped, no remainder
   1. Prerequisites
      a. Solves division problems for dividends to 100.
      b. Division is distributive over addition and subtraction.
   2. Behaviors to be taught
      a. Estimates answers in division by rounding division, divided, or both.
      b. Uses distributive principle to simplify division problems for two or three digit products.
      c. Solves division word problems.
      d. Does short division.
   3. Enrichment
      a. None

K. Subtraction (fractions) - inverse of adding fractions
  1. Prerequisites
    a. Shows subtraction is inverse of addition.
    b. Adds or subtracts any two fractions with equal denominators.
    c. Column addition of fractions — equal or unequal denominators.
  2. Behaviors to be taught
    a. Adds, subtracts, or multiplies fractions with unequal denominators.
    b. Solves one step word problems involving fractions to twelfths.
    c. Adds or subtracts improper and mixed fractions.
  3. Enrichment
    a. None

L. Subtraction of fractions — property of zero.
  1. Prerequisites
    a. Subtraction is not commutative.
    b. Adds or subtracts fractions with equal denominators.
  2. Behaviors to be taught.
    a. Behavior analysis in progress.
  3. Enrichment
    a. None

M. Expanded notation for fractions
  1. Prerequisites
    a. Writes numerals in expanded notation.
    b. Identifies place value — numbers to 10,000.
  2. Behaviors to be taught
    a. Reads decimal numbers.
    b. Charts decimal numbers
    c. Reads decimal fractions.
  3. Enrichment
    a. None

N. Empty set
  1. Prerequisite behaviors
    a. Writes proper numeral when shown a set in N notation.
    b. Behavior analysis in progress.
  2. Behaviors to be taught
    a. Identifies cardinal number.
    b. Behavior analysis in progress.
  3. Enrichment
    a. None

O. Read and write to one hundred billion
  1. Prerequisites
    a. Identifies the number of tens and ones in pictures.
    b. Writes numeral in expanded notation form.
  2. Behaviors to be taught
    a. Writes numeral in expanded notation form.
    b. Identifies place value.
    c. Writes standard numeral for a number represented as words.
    d. Reads numbers to trillions.
  3. Enrichment
    a. None

P. Recognize two-dimensional figures.
1. Prerequisites
a. Can identify a plane.
b. States or demonstrates what a plane is using points.
2. Behaviors to be taught
a. Identifies intersecting lines.
b. Identifies a plane
c. Discriminates between lines in a plane and lines not in a plane.
3. Enrichment
a. None

Q. Recognize three-dimensional figures
1. Prerequisites
a. Recognizes names of geometric figures.
b. Can identify geometric figures.
2. Behaviors to be taught
a. Knows and can select geometric figures or objects from a group of object
b. Finds surface areas of objects.
3. Enrichment
a. None

II. Items to be introduced
A. Addition with common fractions notation, unequal denominators.
1. Prerequisites
a. Adds or subtracts fractions with equal denominators.
b. Renames fractions in lowest terms.
2. Behaviors to be taught
Adds, subtracts, or multiplies fractions with unequal denominators.
a. Solves one-step word problems-fractions to twelfths.
b. Finds least common denominator.
3. Enrichment
a. None
B. Subtraction-fractions-unequal denominators
1. Prerequisites
a. Pittsburgh University Center - has content item.

C. Commutativity - multiplication - fractions
1. Prerequisites
a. Uses the commutative property of multiplication.
b. Uses the commutative and associative principles in adding fractions.
2. Behaviors to be taught
a. Multiplies fractions with factors of up to 4/4 by whole numbers to 20.
b. Performs addition, subtraction, or multiplication with fractions with unequal denominators.
3. Enrichment
a. None

D. Multiplication of fractions developed from the number line.
1. Prerequisites
a. Adds or subtracts fractions equal denominators.
b. Adds two numbers with whole numbers parts and one or two decimal places.
2. Behaviors to be taught
a. Multiplies fractions with factors of up to 4/4 by whole number factors up to 20.
b. Uses picture regions, number lines, etc. when doing simple operations with unequal denominators.
3. Enrichment
a. None

E. Multiplication-fractions-identity element
    1. Prerequisites
        a. Uses x sign in multiplication.
        b. Shows 1 is identity element for multiplication.
    2. Behaviors to be taught
        a. Understands dividing or multiplying fraction numerator or denominator by same number does not change fraction.
        b. Can provide a fraction for a number pair.
        c. Identifies other names for numbers.
    3. Enrichment
        a. None

F. Infinite sets
    1. Prerequisites
        a. Identifies a set of whole numbers as a set beginning with zero.
        b. Writes proper numeral when showing a set in N notation.
    2. Behaviors to be taught
        a. List multiples of any number.
        b. Uses braces to enclose members of a set.
    3. Enrichment
        a. None

G. Meaning of set and member
    1. Prerequisites
        a. Can identify meaning of word set.
    2. Behaviors to be taught
        a. Identifies subset
        b. Behavior analysis in progress.
    3. Enrichment
        a. None

H. Finite set
    1. Prerequisites
        a. Identifies cardinal number of a group.
        b. Writes proper numeral when shown set in N notation.
    2. Behaviors to be taught
        a. Writes proper numeral when shown a set in N notation.
    3. Enrichment
        a. None

III. Optional Items
    A. Powers and exponents
    B. Intersection (Venn diagrams)
    C. Volume
    D. Averages
    E. Union of sets
    F. Measuring (linear)
    G. Liquids
    H. Rounding numbers
    I. Number sequences
    J. Percent
    K. Ratio
    L. Finding perimeter
    M. Money (crossword puzzle)
    N. Area II

Grade Six
Primes Math

I. Items to be mastered
  A. Division of fractions the inverse of multiplying
    1. Prerequisite behaviors
      a. One is the identity element
      b. Uses the multiplication algorithm
    2. Behaviors to be taught
      a. Identifies multiplying or dividing the numerator and denominator by the same number as being equivalent to multiplying or dividing the fraction by one.
      b. Uses the division algorithm

  B. Multiplication of fractions developed from addition of equal fractions.
    1. Prerequisite behaviors
      a. Solves multiplication problems for products to 200
      b. Adds or subtracts any two fractions with equal denominators.
    2. Behaviors to be taught
      a. Multiplies a proper fraction by a whole number

  C. Commutativity - multiplication - fractions
    1. Prerequisite behaviors
      a. Completes multiplication equation which illustrate commutative property.
      b. Uses the commutative and associative principles in addition of fractions.
      c. Identifies the relationship "1/A or N"
    2. Behaviors to be taught
      a. Solves problems with zero in a factor
      b. Applies the associative and commutative principle in multiplication.
      c. Uses the multiplication algorithm
      d. Behavioral analysis in progress

  D. Addition - fractions - binary - operation
    1. Prerequisite behavior
      a. Adds or subtracts any two fractions with equal denominators.
      b. Uses the commutative and associative principles.
      c. selects those number statements which illustrate a principle
    2. Behaviors to be taught
      a. There are no behaviors for this item

  E. Addition - fractions - unequal denominators
    1. Prerequisite behaviors
      a. Adds or subtracts any two fractions with equal denominators.
      b. Uses the algorithm for addition and subtraction of fractions.
      c. Performs column addition with two or more simple fractions.
    2. Behaviors to be taught
      a. Performs complex addition and subtraction.
      b. Finds the least common multiple
      c. Performs simple addition, subtraction or multiplication with fractions unequal denominators.

F. Decimal fraction notation
   1. Prerequisite behaviors
      a. Completes problems which illustrate the commutative or associative property
      b. Adds two or more addends
   2. Behaviors to be taught
      a. Adds two numbers with whole number parts.
      b. Adds two or more numbers with whole number parts.

G. Division of fractions: property of one
   1. Prerequisite behaviors
      a. Finds the greatest common factor.
      b. Uses the multiplication algorithm for multiplying all fractions.
   2. Behaviors to be taught
      a. Identifies multiplying or dividing the numerator and denominator by the same number as being equivalent to multiplying or dividing the fraction by one.

H. Common fraction notation
   1. Prerequisite behaviors
      a. Performs simple addition, subtraction or multiplication with unequal denominators.
      b. Multiplies a proper fraction by a whole number.
      c. Uses the multiplication algorithm for multiplying fractions.
   2. Behaviors to be taught
      a. Applies the associative or commutative principle in multiplication.
      b. Uses the multiplication algorithm.
      c. Performs more complex multiplication of fractions.
      d. Multiplies a proper fraction by a whole number. Reduces answer to lowest terms.

I. Equivalent common fractions
   1. Prerequisite behaviors
      a. Uses objects, pictured objects or no pictured objects, to identify different fractional statements.
      b. Identifies one as the identity element for multiplication.
      c. Multiplies a proper fraction by a whole number.
   2. Behaviors to be taught
      a. Identifies other names for numbers.
      b. Finds a set of equivalent fractions for a given fraction.
      c. Selects or writes fractions which depict a certain ratio or proportion.
      d. Performs simple ratio or proportion problems.

J. Ordering-fractions
   1. Prerequisite behaviors
      a. Supplies the correct sign
      b. Uses the number line
      c. Performs simple ratio and proportion problems.
   2. Behaviors to be taught
      a. Reads decimal fractions to millionths
      b. Supplies the correct sign $>$ $<$ or =
      c. Adds or subtracts fractions and whole numbers.
      d. Finds a set of equivalent fractions for a given fraction.

- K. Subtraction of fractions developed in relation to subsets.
    - 1. Prerequisite behaviors
        - a. Uses set braces to enclose members.
        - b. Identifies a subset of a given set by separating the set.
        - c. Identifies a subset by separating a specified number of sets by partitioning
    - 2. Behaviors to be taught
        - a. Not in the computer printout.

II. Introductory items

- A. Division - fractions - binary operation
    - 1. Prerequisite behaviors
        - a. Uses the division algorithm
        - b. Identifies which number systems have closure property.
        - c. Identifies the associative, commutative, distributive principles.
    - 2. Behaviors to be taught
        - a. Not in computer printout.

- B. Noncommutativity, nonassociativity of division (fractions)
    - 1. Prerequisite behaviors
        - a. Applies commutative principle in multiplication of fractions.
        - b. Applies associative principle in multiplication of fractions.
    - 2. Behaviors to be taught
        - a. Performs addition, subtraction, multiplication or division for problems involving any common fractions.
        - b. Identifies the associative, commutative and distributive statements.
    - 3. Enrichment
        - a. Worksheets on the principles.

- C. Division of fractions closure
    - 1. Prerequisite behavior
        - a. Uses the division algorithm for dividing proper, improper, complex fractions.
        - b. Identifies which operations have the closure.
    - 2. Behaviors to be taught
        - a. Not in computer printout.
    - 3. Enrichment
        - a. Worksheets on the closure property.

- D. Distributivity - fractions
    - 1. Prerequisite behaviors
        - a. Shows the distributive property of multiplication over addition and subtraction.
        - b. Uses the multiplication algorithm.
        - c. Multiplies a decimal number times a decimal number.
    - 2. Behaviors to be taught
        - a. Applies the distributive principle.
        - b. Uses the multiplication algorithm.for multiplying all fractions.
    - 3. Enrichment
        - a. None

- E. Product sets to form new sets.
    - 1. Prerequisite behaviors
        - a. Behavioral analysis in progress.
    - 2. Behaviors to be taught
        - a. Behavioral analysis in progress.

F. Partitioning sets to form new sets
  1. Prerequisite behaviors
    a. Uses set braces to enclose members.
    b. Selects a set which is a proper subset of a given set.
  2. Behaviors to be taught
    a. Identifies a subset of a given set by separating the set.

G. Zero not a divisor fractions
  1. Prerequisite Behaviors
    a. Completes the inverse problem for division.
    b. Completes the inverse problem for multiplication.
    c. Uses the division algorithm.
  2. Behaviors to be taught
    a. Not in computer printout.

H. Equivalent percent notation
  1. Prerequisite behaviors
    a. Finds a set of equivalent fractions for a given fraction.
    b. Identifies other names for numbers.
    c. Writes the decimal equivalent for simple fractions.
    d. Changes decimal equivalent to fractions.
  2. Behaviors to be taught
    a. Selects or writes fractions which depict a certain ratio or proportion.
    b. Does problems requiring conversion from common fractions to percent.

I. Numbers expressed in exponential form
  1. Prerequisite behaviors
    a. Multiplies numbers in exponential form with same bases and positive powers.
  2. Behaviors to be taught
    a. Not in computer printout
  3. Enrichment
    a. Teacher constructed materials.

J. Graphing - coordinate - plane
  1. Prerequisite behavior
    a. Graphs the answer set of a statement.
    b. Constructs number line and identifies the answer.
  2. Behaviors to be taught
    a. Behavioral analysis in progress.

K. Division of fractions: closure
  1. Prerequisite behavior
    a. Uses the division algorithm.
    b. Identifies a certain principle.
    c. Identifies operations which have closure.
  2. Behaviors to be taught
    a. Not in computer printout.

L. Ordered pairs on a coordinate plane
  1. Prerequisite behavior
    a. Graphs the answer set of a statement with literal variables.
    b. Constructs a number line and identifies the answer on a number line.

2. Behaviors to be taught
    a. Graphs lines and figures on a coordinate graph.
    b. Gives the number pairs in matching two sets.
    c. Makes a table or ordered pairs.
    d. Plots ordered pairs of numbers on graph paper.

M. Construct or draw three-dimensional figures
  1. Prerequisite behaviors.
    a. Measures angles using a protractor.
    b. Draws angles with a protractor.
    c. Discriminates between lines in a plane and lines not in a plane.
  2. Behaviors to be taught
    a. Identifies a triangular pyramid.
    b. Constructs a triangular pyramid.
    c. Identifies the surface area of prisms, cylinders, pyramids, cones.

III. Optional items
  A. Rounding for estimates
    1. The students will use the following materials
      a. Paper
      b. Ditto
      c. Silver-Burdett text

  B. Addition of base five numerals
    1. Materials needed
      a. Tape recorder
      b. Holt text
      c. Ditto

  C. Place value decimals
    1. Materials
      a. Ditto
      b. Place value board

  D. Angles (construction)
    1. Materials
      a. Ditto
      b. Tape recorder
      c. Paper
      d. Silver-Burdett text

  E. Associativity - directions are on the ditto
    1. Materials
      a. Ditto
      b. Paper
      c. Pencil
      d. Tape recorder

  F. Development of standard units - (Group work)
    1. Materials
      a. Ditto
      b. Teaching tape
      c. Silver-Burdett text

G. Temperature
    1. Materials
        a. Text - Silver-Burdett
        b. Teaching tape
        c. Scale-therm

H. English and metric units
    1. Materials
        a. Silver Burdett text
        b. Teaching tape
        c. Ditto

I. Basic concepts of percent
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

J. Describe and analyze geometric figures
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

K. Common fractions, unequal denominators
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

L. Circles
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

M. Probability
    1. Materials
        a. Teaching tape
        b. Houghton - text

N. Clock arithmetic
    1. Materials
        a. Teaching tape
        b. Houghton - text
        c. Clock face

O. Finding perimeter of circumference
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

P. Planes
    1. Materials
        a. Teaching tape
        b. Silver-Burdett text

Q. Ordering of fractions
    1. Materials
        a. Drill tape
        b. Text

R. Commutativity and associativity
    1. Materials
        a. Drill tape
        b. Text-Silver Burdett

S. Historical systems of numeration
    1. Materials
        a. Teaching tape
        b. Houghton and Holt text